

THREE-PHASE STATE ESTIMATION IN POWER DISTRIBUTION SYSTEMS

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Doctoral Dissertation
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TRIFAZNI OCENJEVALNIK STANJA V DISTRIBUCIJSKIH ENERGETSKIH SISTEMIH

Doktorska disertacija

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Abstract

This dissertation investigates the problem of observability in power distribution networks. A recent increase in the share of renewable energy sources has increased uncertainty and variability on the generation part of the power equation. Due to lower nominal powers, these sources are normally connected to distribution networks, which poses technical challenges to reliable network operation. In the dissertation, we first examine the accurate modeling of a power network. We proposed a unified three-phase branch model which can represent all the relevant components of a power distribution network (overhead lines, underground cables, transformers in any connection, tap changers in any connection, and voltage regulators). The presented model enables a straightforward construction of the network-admittance matrix, i.e. the network model. The network model is crucial in the later processing with network observability algorithms. In relation to the model, we have reviewed the methods for obtaining model parameters, which comprise methods for self and mutual series impedances, methods for self and mutual shunt admittances, and the calculation of connection matrices.

Further, we have reviewed various state estimators. Since we are dealing with the estimation of deterministic parameters, the reviewed estimators belong to the so-called class of classic estimators. Maximum Likelihood Estimation (MLE) and Weighted Least Squares (WLS) estimation are discussed in detail. Besides the MLE and WLS estimators, we also discuss estimators that exhibit a robustness or insensitivity to bad data present in the input set of measurements. From this class of robust estimators, the Schweppe Huber Generalized-M (SHGM) estimator and Least Absolute Value (LAV) are discussed in detail. The reviewed estimators are compared based on calculation times and statistical efficiency for different measurement configurations on the reference IEEE distribution feeder.

Further, the dissertation discusses sensitivities to small model and measurement errors. It is frequent in practice that actual conductor lengths differ from the modeled ones. The conducted sensitivity analysis determines the uncertainty intervals in the estimated state variables from the uncertainty intervals in the input model variables. For analysis, the approach with perturbation of the Karush-Kuhn-Tucker conditions was leveraged, which yields all the specified sensitivities at once.

In the final part of the dissertation, the application of the state estimation solution to a real-world power distribution network is discussed. Besides the implementation of the algorithm itself, the real-world implementation requires many other support functions architected into a workable solution. The software architecture of the solution and the detailed process of implementation are presented. The dissertation also presents the concept of a low-cost synchrophasor measurement device, suitable for implementation in power distribution networks.

Povzetek

Disertacija obravnava problem spoznavnosti v tri-faznih razdelilnih električnih omrežjih. Povečevanje deleža obnovljivih virov energije in električnih vozil v zadnjih letih povečuje nepredvidljivost in variabilnost generacijskega dela močnostne bilance. Zaradi manjših nazivnih moči se tovrstni viri praviloma prikljapljajo v razdelilna električna omrežja, kar pa prinaša številne tehnične izzive na področju zagotavljanja zanesljivosti obratovanja omrežja. V disertaciji smo se v prvi fazi posvetili točnemu modeliranju omrežja. Razvili smo poenoten tri-fazni model veje električnega omrežja, s katerim je možno enotno predstaviti vse, za naš problem relevantne, gradnike električnega razdelilnega omrežja (nadzemne vode, podzemne kable, napetostne regulatorje v poljubnih vezavah, transformatorje v poljubnih vezavah in transformatorje s spremenljivim odcepom). Predstavljen model omogoča enostavno gradnjo admitančne matrike omrežja t.j. modela omrežja. Model omrežja pa je ključen element pri nadaljni obdelavi z algoritmi za spoznavnost omrežja. V navezavi na model so pregledani postopki za pridobitev parametrov modela, kar obsega izračun medsebojnih in lastnih vzdolžnih impedanc, medsebojnih in lastnih dozemnih admitanc, ter izračun povezavnih matrik.

V nadaljevanju so pregledani različni ocenjevalniki stanja. Ker imamo opravka z iskanjem determinističnih parametrov, obravnavani estimatorji spadajo med t.i. klasične estimatorje. Detajlno sta obravnavani metoda največjega verjetja (angl. Maximum Likelihood Estimation) in metoda uteženih najmanjših kvadratov (angl. Weighted Least Squares). Poleg teh so obravnavane še metode, ki izkazujejo robustnost oziroma neobčutljivost na prisotnost slabih meritev v vhodnem vzorcu meritev. Med njimi sta podrobno obravnavani metoda najmanjše absolutne vrednosti (angl. Least Absolute Value) in splošna M-metoda Scweppeja in Huberja. Za obravnavane metode so podani algoritmi za numerično reševanje vključno s primerjavami hitrosti izračuna in statistične učinkovitosti za različne konfiguracije meritev na referenčnem IEEE razdelilnem omrežju.

V disertaciji smo obravnavali tudi občutljivosti obravnavanih ocenjevalnikov stanja na majhne napake parametrov modela ter majhne napake meritev. Realne dolžine vodnikov v praksi namreč pogosto odstopajo od modeliranih in opravljena analiza za določen odstotek odstopanja pokaže možne intervale odstopanja v izračunanih spremenljivkah stanja in iz njih izvedenih količinah. Za analizo je bil uporabljen pristop perturbacije Karush-Kuhn-Tuckerjevih pogojev, ki omogoča izračun vseh željenih občutljivosti v enem koraku.

Disertacija v zadnjem delu obravnava aplikacijo sistema za ocenjevanje stanja na realnem razdelilnem energetskega omrežju. Implementacija sistema na realnem omrežju poleg implementacije samega algoritma za ocenjevanje stanja zahteva več podpornih komponent, ki morajo biti povezane v delujočo rešitev. Predstavljena je arhitektura implementirane programske rešitve, vključno s tehničnimi podrobnostmi procesa implementacije. V disertaciji je predstavljen tudi koncept nizkocenovne merilne naprave za merjenje sinhrofazorjev, ki je primerna za implementacijo v razdelilnih energetskega omrežjih.

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Abbreviations

AMI	...	Advanced Measurement Infrastructure
API	...	Application Programming Interface
AWG	...	American Wire Gauge
BLUE	...	Best Linear Unbiased Estimator
CRLB	...	Cramer-Rao Lower Bound
DER	...	Distributed Energy Resources
DG	...	Distributed Generation
DSO	...	Distribution System Operator
DSSE	...	Distribution System State Estimation
EP	...	Elektro Primorska
EV	...	Electric Vehicles
FDIR	...	Fault Detection, Isolation, and Service Restoration
GMR	...	Geometric Mean Radius
GUI	...	Graphical User Interface
IPS	...	International Postgraduate School
IRLS	...	Iteratively Re-weighted Least Squares
JSI	...	Jožef Stefan Institute
KKT	...	Karush-Kuhn-Tucker
LAV	...	Least Absolute Value
LOMED	...	Low Median
LS	...	Least Squares
LSE	...	Least Squares Estimation
MC	...	Monte Carlo
MLE	...	Maximum Likelihood Estimation
MVUE	...	Minimum Variance Unbiased Estimator
MQTT	...	Message Queuing Telemetry Transport
PDF	...	Probability Density Function
PLC	...	Power Line Communications
PMU	...	Phasor Measurement Unit
SE	...	State Estimation
SG	...	Smart Grid
SHGM	...	Schweppe-Huber Generalized-M
SM	...	Smart Meter
TSO	...	Transmission System Operator
WLS	...	Weighted Least Squares

Symbols

Θ_m^a	... Voltage phase angle at node m on phase a
$ V_m^a $	... Voltage magnitude at node m on phase a
E_k^a	... Bus voltage phasor
I_{km}^a	... Line current phasor
P_{km}^a	... Active line power flow
Q_{km}^a	... Reactive line power flow
S_{km}^a	... Complex line power flow
P_k^a	... Active bus power injection
Q_k^a	... Reactive bus power injection
R	... Resistance
X	... Reactance
C	... Capacitance
z_{km}^{ij}	... Conductor series impedance element between buses k and m
r_{km}^{ij}	... Conductor series resistance element between buses k and m
x_{km}^{ij}	... Conductor series reactance element between buses k and m
$y_{km,sh}^{ij}$	... Conductor shunt admittance element between buses k and m
$g_{km,sh}^{ij}$	... Conductor shunt conductance element between buses k and m
$b_{km,sh}^{ij}$	... Conductor shunt susceptance element between buses k and m
G_{km}^{ij}	... System-admittance conductance element
B_{km}^{ij}	... System-admittance susceptance element
t_{km}^{ij}	... Connection element between buses k and m , on the side of bus k
J	... Objective function of the state estimator
J^*	... Objective function of the state estimator at the optimal solution
$\hat{z}_{ij}$	... Primitive series impedance element
$\hat{P}_{ij}$	... Primitive potential coefficient element
f_i	... Measurement i
x_i	... State variable i
h_i	... Measurement function i
r_i	... State estimation residual
c_i	... State estimation equality constraint
g_i	... State estimation inequality constraint
γ_i	... SHGM estimator weight
ρ	... SHGM estimator function
ξ_i	... number of nonzero entries in the i -th row of the Jacobian matrix
ϵ_0	... Permittivity the free space
ϵ_r	... Relative permittivity of the medium
p	... Probability density function
GMR_i	... Geometric Mean radius of conductor i in feet
D_{ij}	... Distance between conductors i and j in feet
S_{ii}	... Distance from conductor i to its image in the ground

S_{ij}	... Distance from conductor i to image in the ground of conductor j
RD_i	... Radius of conductor i
d_c	... Phase conductor diameter
d_{od}	... Nominal diameter over the concentric neutrals of the cable
d_s	... Diameter of a concentric neutral strand
r_c	... Resistance of the phase conductor
r_s	... Resistance of a solid neutral strand
k	... number of concentric neutral strands
T	... thickness of copper tape shield
w_i	... Noise contribution to measurement i
m	... Number of measurements
n	... Number of state variables
o	... Number of equality constraints
l	... Number of inequality constraints
σ_i	... Standard deviation of measurement noise contribution i
Ω_p	... Set of phases
Ω_I	... Set of active inequality constraints
Λ	... Set of buses adjacent to bus k including bus k
$\Theta(\lambda, \mu)$	... Dual function of the optimization problem
$\kappa(\mathbf{A})$	... Condition number operator
α	... Hachtel's scaling factor
$\mathcal{L}$	... Lagrangian function
$\mathcal{N}$	... Normal probability density function
$\mathbf{f}$	... Vector of measurements
$\mathbf{g}$	... Function vector
$\mathbf{x}$	... Vector of state variables
$\mathbf{h}$	... Vector of measurement functions
$\mathbf{z}$	... Vector of measurements
$\mathbf{r}$	... Vector of residuals
$\mathbf{c}$	... Vector of equality constraints
$\mathbf{g}$	... Vector of inequality constraints
$\mathbf{a}$	... Vector of measurements and model data that are used in the calculation of sensitivities
$\mathbf{I}_{km}^{a,b,c}$	... Vector of branch currents
$\mathbf{E}_k^{a,b,c}$	... Vector of bus voltages
$\mathbf{P}_i$	... Vector of active power injection measurements in the system
$\mathbf{Q}_i$	... Vector of reactive power injection measurements in the system
$\mathbf{P}_{ij}$	... Vector of active power flow measurements in the system
$\mathbf{Q}_{ij}$	... Vector of reactive power flow measurements in the system
$ \mathbf{V} $	... Vector of voltage magnitude measurements in the system
Θ	... Vector of voltage phase measurements in the system
$\hat{\mathbf{x}}$	... Estimated vector of state variables
$\mathbf{w}$	... Vector of noise contributions to measurements
λ	... Vector of Lagrange multipliers for equality constraints
μ	... Vector of Lagrange multipliers for inequality constraints
$\mathbf{x}^*$	... Vector of state variables at the optimal solution
λ^*	... Vector of Lagrange multipliers for equality constraints at the optimal solution
μ^*	... Vector of Lagrange multipliers for inequality constraints at the optimal solution
$\mathbf{R}$	... Measurement covariance matrix
$\mathbf{I}$	... Identity matrix

$\mathbf{0}$	... Null matrix
$\hat{\mathbf{Z}}_{km}$	... Primitive series impedance matrix
$\hat{\mathbf{P}}_{km}$	... Primitive potential coefficient matrix
$\mathbf{C}$	... Matrix of first order partial derivatives of equality constraints
$\mathbf{C}_{abc}$	... Capacitance matrix
$\mathbf{Z}_{km}$	... Series impedance matrix between buses k and m
$\mathbf{B}$	... System conductance matrix
$\mathbf{G}$	... System susceptance matrix
$\mathbf{Y}_{km,sh}$	... Shunt admittance matrix between buses k and m
$\mathbf{Y}_{b,km}$	... Branch admittance matrix between buses k and m
$\mathbf{T}_{km}$	... Connection matrix between buses k and m on the side of bus k
$\mathbf{T}_{mk}$	... Connection matrix between buses m and k on the side of bus m
$\nabla_{\mathbf{x}}$	... Gradient with respect to vector $\mathbf{x}$
$\nabla_{\mathbf{xa}}$	... Hessian with respect to vectors $\mathbf{x}$ and $\mathbf{a}$ respectively

Chapter 1

Introduction

In the past decade, it has become apparent that global climate changes are man-caused phenomena. Consequently, a vast societal consensus for a shift to low carbon energy sources and transportation has emerged [1]. This gave rise to government incentives for the installation of renewable generation plants, such as wind and solar power plants. Consequently, a large number of small-size units (in terms of nominal power) have been installed and connected to the distribution grid. This invalidates the traditional view of power systems, comprised of large-scale generation units connected to the transmission grid, which supply distribution networks, which unidirectionally supply the end customers.

The upcoming years will bring investments in the domain of electric mobility. Currently, the world's transportation sector accounts for more than a quarter of total energy consumption and is, almost exclusively, fueled by oil products. The most natural point of connection of Electrical Vehicles (EVs) will again be distribution systems. Besides EVs, large-scale energy storage devices are expected to be deployed throughout the network.

All these changes have the potential to markedly reduce the total carbon footprint. However, they also significantly increase the system complexity which makes its operation to ever stricter reliability margins even harder.

The described context gave rise to the concept of Smart Grid (SG) as the modernized power network in which advanced automated functions are implemented, exploiting a suitable communication infrastructure to efficiently perform the management, control, and protection of the network [2].

To effectively control and manage the assets in the network, an accurate picture of the current operating conditions must be obtained. Such information is extracted from the measurement system. The measurement system comprises of measurement devices deployed across the network, as well as all processing and collection algorithms and software.

The State Estimation (SE) solution, which enables full system observability with the required accuracy and resolution, is an essential part of the measurement system. SE is a signal processing technique designed to extract values of a group of parameters (state variables) from a collection of noisy measurements. In the case of power systems, the selected state variables are normally bus voltage phasors or branch current phasors. Once the state variables are obtained, it is possible to calculate any quantity of interest and run any control or management function.

SE is expected to play a key role in distribution networks, which makes it an important research topic and a challenging task. This dissertation aims to address some of the issues related to the design and implementation of such a system in power distribution networks.

1.1 Purpose of the Dissertation

The general purpose of this dissertation is to overcome the challenges of the state of the art in state estimation methods for distribution power systems. More specifically, the purpose of the dissertation is to develop, implement, and evaluate a feasible three-phase distribution system state estimation along with a configuration of measurement devices, which would satisfy different application needs, such as coordinated voltage control, estimation of line impedances, etc.

1.2 Hypothesis and Goals of the Dissertation

The main goal of this dissertation is to develop, implement, and thoroughly evaluate a three-phase Distribution System State Estimation (DSSE). More specifically, we aim to achieve the following goals:

- To present a unified branch model that allows a unified representation of three-phase distribution network elements. To present conductors, transformers, tap changers, and voltage regulators in any connection, with the same model, from which the branch-admittance matrices can then be derived. To represent voltage regulators and tap changers as a single branch and, in comparison with existing approaches, eliminate the need to introduce an additional bus.
- To develop a three-phase DSSE model that works with heterogeneous measurement types (active and reactive power injections, branch currents, active and reactive power flows, and bus voltages) and is suitable to handle large distribution networks in near real-time.
- To suggest a high accuracy Phasor Measurement Unit (PMU) concept suitable for deployment in networks with low reactance to resistance ratios, i.e. power distribution networks.
- To evaluate the impact of model and measurement uncertainties for different measurement configurations and different state estimation algorithms. To implement a method that evaluates all of the sensitivities at once, and is feasible for the evaluation of large distribution networks.
- To develop and operationally deploy a state estimation system as well as the suggested high-accuracy PMU in a real-world network.

1.3 Contributions

In the dissertation, we investigated the problem of observability in power distribution networks. The results presented in this dissertation are expected to contribute to further research and the development of methods, models, procedures, and algorithms for SE in distribution networks. In the dissertation, we claim the following main contributions:

- Development of a unified three-phase branch model that allows a unified representation of the three-phase distribution network elements. This contribution is presented in Chapter 2 and was published in [3].
- Development and implementation of a three-phase DSSE that is able to accommodate heterogeneous measurement types and handle real-world distribution networks. This contribution is presented in Chapter 3 and was published in [4].

- Sensitivity analysis of different three-phase SE algorithms to uncertain line lengths and measurements is performed. Guidelines for the design of measurement configurations that can reduce the impact of uncertainties of model and measurements in the three-phase DSSE are given. This contribution is presented in Chapter 4 and was published in [5].

Further contributions that are also of high importance to the researches working in the area include the following:

- An upper error bound in interval sensitivity analysis is suggested.
- A concept of high accuracy Phasor Measurement Unit (PMU) suitable for distribution networks is proposed.

1.4 Organization of the Dissertation

The dissertation is organized according to the following outline. In Chapter 2, the evolution of power distribution networks and the need for observability are described. Further, SE is introduced as a solution and compared to the SE in transmission networks. The most relevant differences between the two systems are outlined. Next, the problem of SE is defined mathematically and different estimators (solutions) are reviewed. Finally, the complete process of modeling the distribution network for the purpose of SE is reviewed.

In Chapter 3, the numerical solution algorithms for the reviewed estimators are presented. Different numerical algorithms are compared, and considerations with respect to condition numbers, measurement configurations, speed of computation, and statistical efficiency are given.

Chapter 4 investigates the impact of model and measurement uncertainties on the state estimation results for different estimators and different measurement configurations in a three-phase distribution network. To obtain all the sensitivities of interest, the approach with a perturbation of the Karush-Kuhn-Tucker conditions is implemented. The sensitivities obtained are expressed in terms of uncertainty intervals rather than just partial derivatives. An upper error bound is proposed on the interval analysis.

In Chapter 5, the application of the SE system to a section of the real-world network is described. This chapter describes the complete software architecture comprising of different components such as measurement pre-processing, communication protocols, and measurement storage. The concept of a highly accurate, low-cost PMU device suitable for distribution networks is also presented in this chapter.

Chapter 6 provides final comments and considerations on the obtained results and concludes this dissertation.

Chapter 2

State Estimation

Estimation theory can be found at the heart of many signal processing systems designed to extract the values of a group of parameters from noisy measurements. Statistical State Estimation (SE) was first applied to power systems back in 1970 by Schweppe and others. It was proposed for the real-time monitoring of electric power transmission networks [6]–[8]. The proposed approach quickly gained traction and has since remained an extremely active research area. The so-called static single-phase Weighted Least Squares (WLS) estimation of power transmission networks is now the standard in the electric power industry. It is adopted by virtually every Transmission Network Operator (TSO).

SE was identified as a tool that can detect and reduce the errors in measurement devices and topology models. Measurements are always subject to errors due to limited accuracy, aging, and malfunctioning of devices. Moreover, communication errors can occur, and in cases where information on a specific bus is obtained from a single measurement device, the network operator loses all information about that bus when, for whatever reason, measurements become unavailable.

Although the term SE in general refers to an algorithm or mathematical expression that extracts some predetermined parameters of interest from noisy measurement data, in terms of power systems, SE represents a comprehensive system with a number of additional functionalities besides the core estimation algorithm. These functionalities may vary from system to system, but most commonly they include the following:

- *Topology processor*: builds a network topology (namely system admittance matrix) based on the latest information on the states of switches, circuit breakers, transformer taps, and regulator settings.
- *Observability analysis*: is a pre-calculation step which makes sure the system is observable, i.e. there are enough real-time and pseudo measurements that the SE algorithm will converge on a solution.
- *SE algorithm*: is the core of the SE system. It takes all the provided data (real-time measurements, pseudo-measurements, and topology) and estimates the most probable state of the system in terms of predetermined criteria.
- *Detection of bad data and topology errors*: is a pre-calculation or post-calculation step that determines if a measurement or a network component status (e.g. circuit breaker) has a value that is not likely to be correct or in the expected range.
- *Database with pseudo-measurements*: holds the pseudo-measurement values relevant to specific points in time. This data is normally obtained by means other than just measurement devices. E.g. historical measurements are fed into the machine learning

system which can determine the most probable current values, taking into account calendar data, weather data, etc.

- *Simulation and Contingency Analysis*: is a post-calculation step that enables the planning of contingency scenarios given the current trends in a power system. This enables the operator to anticipate problems and act before they occur.

Fig. 2.1 depicts the relationships of the listed functionalities, and the propagation of data through them.

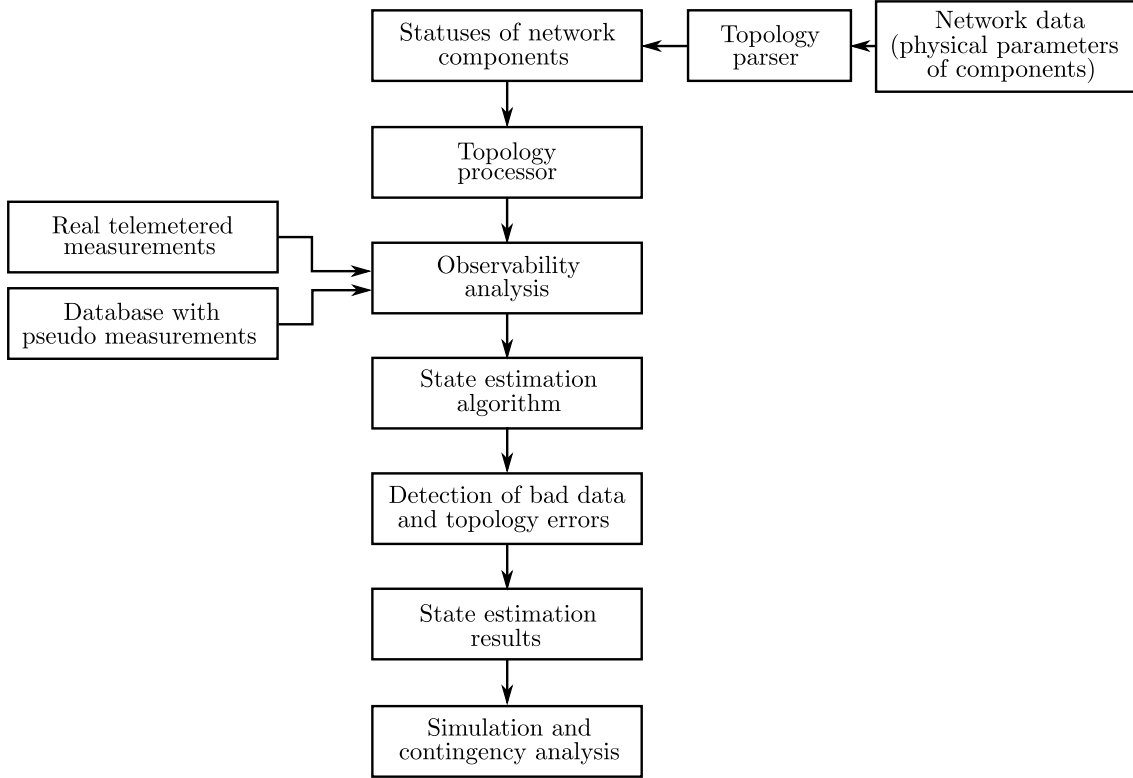


Figure 2.1: Main components of a state estimation system.

2.0.1 Evolution of power distribution systems and the need for implementation of state estimation

Historically, distribution grids were designed for unidirectional power flow. The assumption was that a part of distribution network is connected to the transmission network, and then power flows from that connection point through a medium voltage feeder into low-voltage networks and to customers. Distribution networks were built in a radial topology. Often, the medium voltage feeder operates in a disconnected loop, which ensures a quick restoration of power in case of a single branch outage in that loop. As opposed to transmission networks, the cost of the meshed topology would be higher than is the cost of outage of a single or a couple of entities in the network. Radial topology and the assumption of unidirectional power flow also greatly simplify the design and operation of the network. Loads can be estimated and predicted quite well from urban development plans, and using load-flow calculation, the voltage profile can be determined along the feeder.

In recent years, a massive increase of Distributed Generation (DG), such as wind and solar plant installments, has been witnessed [9]. In most cases, DGs are connected into

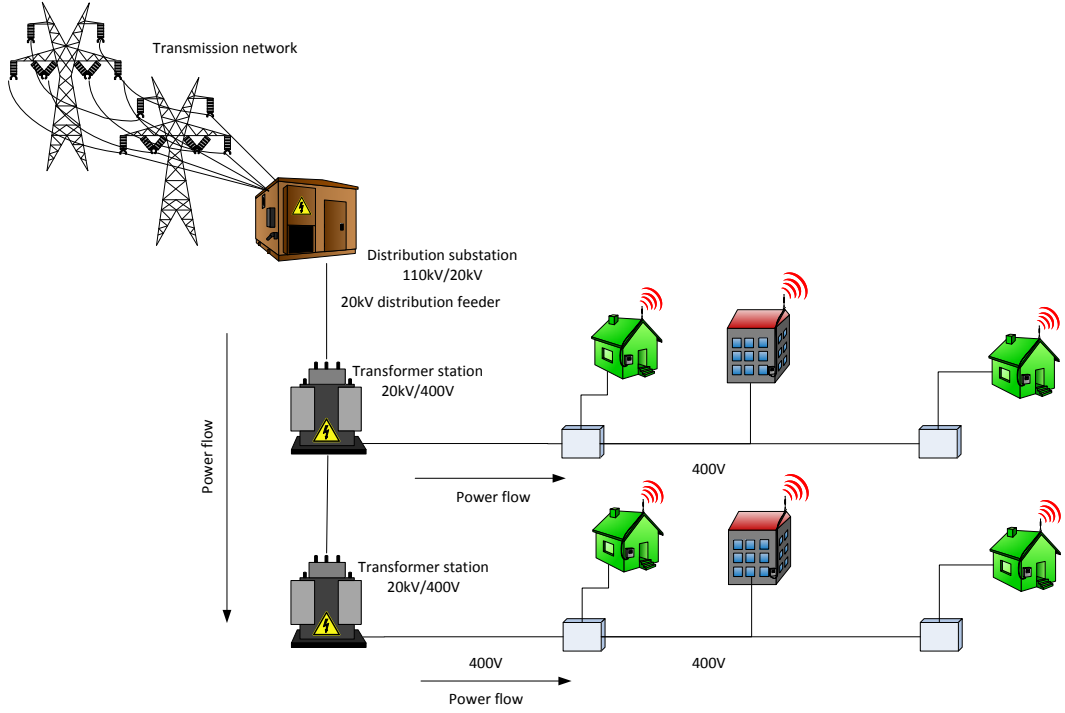


Figure 2.2: Power distribution networks in the past.

distribution networks because the nominal powers are relatively small ($\approx 10 - 10^3$ times the power of an average consumer, as opposed to a conventional power plant that is $\approx 10^4 - 10^7$ times the power of an average customer). Another fundamental change that is underway is the adoption of electric vehicles (EV) [10], [11]. Their spread is expected to have a major impact on distribution systems [12], since the power of the inverter and the battery capacity are significant (orders of $\approx 10kW$ and $\approx 70kWh$ respectively). Since an EV is essentially a battery that is connected to the distribution grid, it can, in general, perform as a load or as a generator. Both DGs and EVs are commonly referred to as Distributed Energy Resources (DERs).

The adoption of DERs is causing bi-directional power flows in distribution systems, which the systems were not designed for. In turn, bi-directional power flows (see Fig. 2.3) can cause under- or over-voltage events, etc. [13], [14]. Since the generation of DERs is irregular and intermittent, it is almost impossible to combine it with the conventional generation. The ramp rates of conventional power plants (such as coal and nuclear) are far too inflexible to allow for rates that are required by renewables [15]. As the conventional generation is being shut down, another significant phenomenon is occurring, namely the reduction of inertia. In order to ensure a continuous stable operation, the power system needs to be able to cope with a high, sudden loss of generation. Battery storage can provide frequency control reserve [16]–[19], but there is still a need for coordinated control of such assets in the network.

The first step towards controllability of the distribution system is observability, i.e. obtaining accurate knowledge about the relevant parameters of the system (see Fig. 2.4). SE will play a crucial role [20] in providing observability for many of the advanced control functions, such as distributed voltage control [21]–[26], load estimation, fault detection,

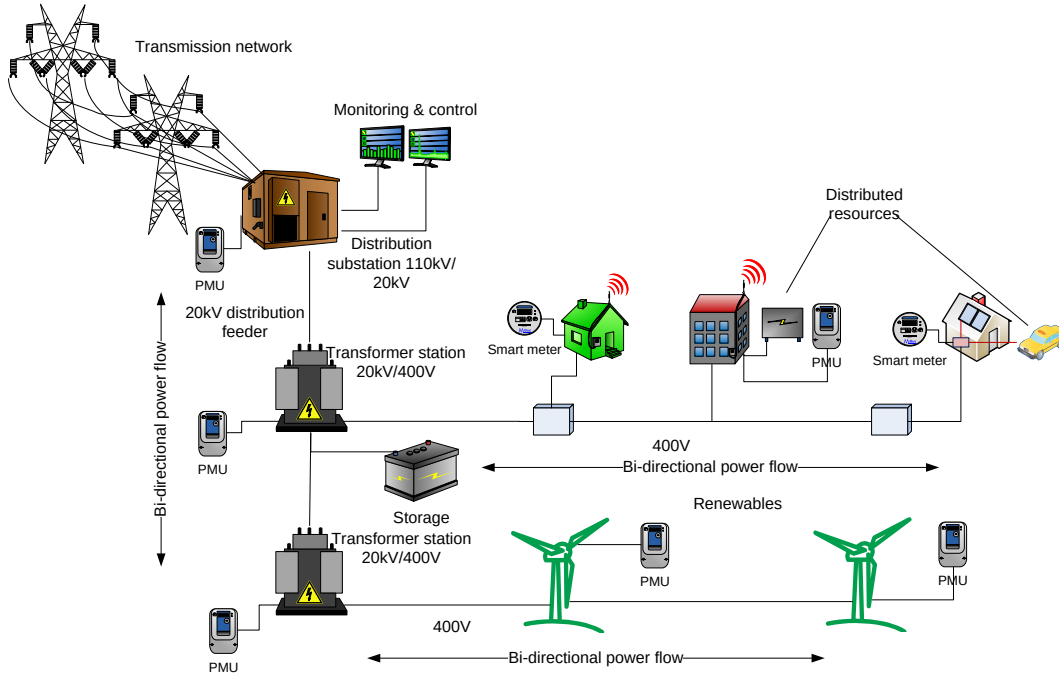


Figure 2.3: Evolution of power distribution networks.

isolation, and service restoration (FDIR), short-circuit analysis, and many others.

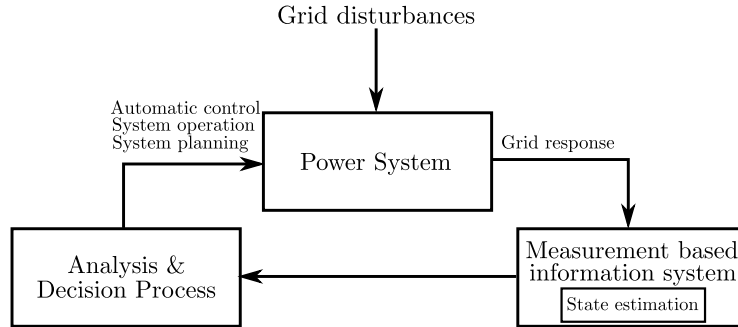


Figure 2.4: Placement of state estimation in the regulation loop.

2.0.1.1 Challenges in distribution systems

Conceptually, SE has the same function in both transmission and distribution systems, but due to some inherent differences between the two, SE for transmission systems cannot be straightforwardly applied to distribution systems. In the case of SE application, the most relevant differences between the two systems are [27]:

- *Imbalance*: A power grid is a three-phase network. In transmission grids, quantities in phases are assumed to be balanced, because of the averaging effect of many number of consumers and transformer connections which can balance their loading

to some extent. Transmission networks are modeled through an equivalent positive sequence phasor in symmetric components. On the contrary, distribution grids extend directly to consumers, thus, single-phase loads and unsymmetric loading are reflected much more in the network. In order to obtain the accurate state of the system, the model needs to account for possible unsymmetries and, thus, all three phases need to be modeled. Most of the approaches in the literature [28]–[32] use the so-called equivalent measurements approach, where all measurements are converted into current measurements. This approach, however, requires an extra computational step. Thus, in the dissertation, we propose a model for the unified representation of three-phase distribution network elements [3]. Conductors, transformers, tap changers, and voltage regulators, in any connection, are all presented with the same model, from which the branch-admittance matrices are derived. With the proposed model, voltage regulators or tap changers and conductor sections or transformers connected in series can be represented as a single branch.

- *Branch parameters:* An electric line has four parameters which affect its ability to fulfill its function as part of a power system, namely series resistance and inductance, and shunt capacitance and conductance [33]. Since the power flows and voltage levels of distribution and transmission networks are different, the conductors have different properties. In transmission networks, the series impedance has a prevailing inductance term. In terms of reactance (X) to resistance (R) ratio (X/R) the ratio is around 10 in transmission networks. This allows for the so-called decoupled estimation, where estimation of voltage magnitudes and angles is separated [34]. Due to relatively high reactance term, phase angle spread is also greater along the line. This makes phase angle differences easier to measure. In distribution networks, the X/R ratios are significantly lower (in the order of 0.1 – 1). As a result, the branch model cannot be simplified to allow for fast decoupled estimation. Also, since the phase angle spread along the line is much smaller, in the case of phase angle measurements, there is a need for very high accuracy measurements.
- *High number of nodes:* Distribution systems commonly have ratios in the order of $10^4 - 10^6$ buses, as opposed to transmission networks that normally have ratios in the order of 100 buses. This number is further multiplied by ≈ 3 since all three phases need to be modeled in the distribution network. This characteristic poses two separate problems. The problem dimension corresponds to the number of buses. Most solution techniques have a computational complexity of $O(n^3)$. The characteristic that can be exploited in addressing this problem is that distribution networks mostly have radial topology. The matrices are, thus, very sparse and data dependency is low. This offers a great opportunity for the parallelization of the solution algorithm. The other challenge to overcome is the low number of measurement devices. Distribution networks are currently poorly monitored. There is little presence of measurement instruments in medium and low voltage networks. In the European Union (EU), there is an effort underway to equip all households with Advanced Measurement Infrastructure (AMI) Smart Meters (SM). For measurements deeper in the network, the crucial factor is price. Because of the large number of nodes, the redundancy of measurements is hard to achieve. One technique to mitigate this issue are so-called pseudo-measurements. These are based on historical data and mathematical models [35], [36]. However, the accuracy of this data is orders of magnitude worse than that of real measurements.
- *Network model uncertainty:* network model parameters such as series impedances or shunt admittances (self and mutual) that are derived from topology data (conductor

geometry) can be incorrect. This is due to inaccurate data provided by the manufacturer or installation crew, inaccurate measurement campaigns, effects of aging, etc. Uncertain model data has influence over the estimated state variables. Several methods were applied to the problem of sensitivity analysis in power system state estimation. In this dissertation, we perform the sensitivity analysis of the state estimation results to small deviations from model or measurement assumptions. Deviations up to a few tens of percent are considered as small. An example of such deviation would be an incorrectly modeled length of a conductor.

2.1 Problem Statement

Mathematically, the state estimation problem can be stated by extracting an unknown vector of system states $\mathbf{x} = [x_1, x_2, \dots, x_n]$ from the m -point dataset $\{f_1, f_2, \dots, f_m\}$ that depends on vector. Thus, to determine $\mathbf{x}$ based on the data, an estimator is defined as:

$$\hat{\mathbf{x}} = \mathbf{g}(\{f_1, f_2, \dots, f_n\}) \quad (2.1)$$

where $\mathbf{g}$ is a function vector that needs to be determined. In the process of estimator determination, the first step is to mathematically model the data. Since the data are inherently random, they are modeled by the Probability Density Function (PDF). The PDF is parameterized by the unknown vector of parameters $\mathbf{x}$. As an example, if there is only one data point and x denotes the mean, then the PDF of the data may be:

$$p(f_1; x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{1}{2\sigma^2}(f_1 - x)^2\right]. \quad (2.2)$$

Since the value of x affects the probability of f_1 , it is possible to infer the value of x from the observed value of f_1 . This specification of the PDF is critical in determining an estimator that will be optimal in a certain sense. Estimation based on PDFs is termed *classical* estimation in which the parameters of interest are assumed to be *deterministic* but unknown [37].

There are different classes of estimators that can be derived for a particular estimation problem. In the case of classical approach, Fig. 2.5 depicts the decision flowchart including different estimators, and decision logic that leads to their selection. In our case, we will first consider the case where we assume that measurement errors have a Gaussian distribution. Considering the flowchart for the selection of the estimator class, we will select the Maximum Likelihood Estimation (MLE). Further, it will be showed that if the assumption about the PDF of measurement errors is dropped, the estimator obtained from the MLE is actually the Weighted Least Squares (WLS) estimator.

Besides the optimality criterion, another important aspect of the state estimation algorithm is its robustness. Robustness is defined as insensitivity of the state estimates to major deviations in a limited number of redundant measurements [38]. In this chapter, two estimators from the class of robust estimators, namely the Least Absolute Value (LAV) and Schweppe Huber Generalized-M (SHGM), and their solution methods are reviewed. Robustness concepts, such as outliers and leverage points, and methods for detecting them are discussed in more detail in [38].

2.1.1 Data Model

In this subsection, a data model for state estimation in distribution power networks will be defined.

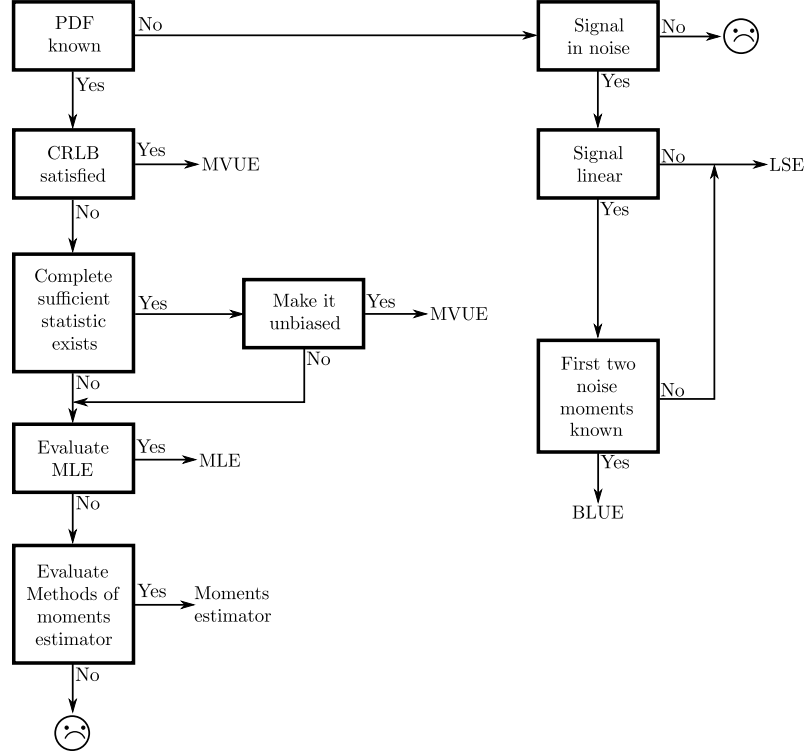


Figure 2.5: Selection of the estimator.

The network state can be inferred from a set of measurements $(f_1, f_2, \dots, f_m)$ that have an established (nonlinear in general) relation to the selected state variables $(x_1, x_2, \dots, x_n)$:

$$\mathbf{f} = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_m \end{bmatrix} = \begin{bmatrix} h_1(x_1, \dots, x_n) \\ h_2(x_1, \dots, x_n) \\ \vdots \\ h_m(x_1, \dots, x_n) \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_m \end{bmatrix} \quad (2.3)$$

$$= \mathbf{h}(\mathbf{x}) + \mathbf{w}$$

The symbol $\mathbf{w} \sim \mathcal{N}(\mathbf{0}, \mathbf{R})$ represents the noise contribution to the measurement vector. It is assumed that the noise in the measurements is uncorrelated and, thus, the measurement covariance matrix is $\mathbf{R} = \text{diag}\{\sigma_1^2, \sigma_2^2, \dots, \sigma_m^2\}$. In section 2.2, measurement functions $h(x_1, \dots, x_n)$ that relate different measurement types with the selected state variables will be developed.

2.1.2 Maximum Likelihood Estimation

The MLE estimator is defined as the value of x (or $\mathbf{x}$) that maximizes the likelihood function. The likelihood function is specified as a PDF. When a closed form expression cannot be found for the MLE, a numerical approach can be utilized for an iterative maximization of the likelihood function.

For the data model defined in subsection 2.1.1, a likelihood function can be written as:

$$\begin{aligned} p(\mathbf{f}; \mathbf{x}) &= \prod_{i=1}^m \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\left[-\frac{1}{2\sigma_i^2}(f_i - h_i(\mathbf{x}))^2\right] \\ &= \prod_{i=1}^m \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\left[-\frac{1}{2}(\mathbf{f} - \mathbf{h}(\mathbf{x}))^T \mathbf{R}(\mathbf{f} - \mathbf{h}(\mathbf{x}))\right]. \end{aligned} \quad (2.4)$$

assuming the differentiability of the likelihood function, the MLE is found from:

$$\frac{\partial \ln p(\mathbf{f}; \mathbf{x})}{\partial \mathbf{x}} = \mathbf{0}. \quad (2.5)$$

Considering Eqn. 2.5, first the logarithm is taken of Eqn. 2.4:

$$\ln p(\mathbf{f}; \mathbf{x}) = \sum_{i=1}^m \frac{1}{\sqrt{2\pi\sigma_i^2}} + \left[-\frac{1}{2}(\mathbf{f} - \mathbf{h}(\mathbf{x}))^T \mathbf{R}(\mathbf{f} - \mathbf{h}(\mathbf{x}))\right]. \quad (2.6)$$

We define a new function $J_{MLE}(\mathbf{x})$ as:

$$J_{MLE}(\mathbf{x}) = \left[-\frac{1}{2}(\mathbf{f} - \mathbf{h}(\mathbf{x}))^T \mathbf{R}(\mathbf{f} - \mathbf{h}(\mathbf{x}))\right]. \quad (2.7)$$

When the derivative of Eqn. 2.4 with respect to $\mathbf{x}$ is taken, Eqn. 2.5 equals:

$$\frac{\partial \ln p(\mathbf{f}; \mathbf{x})}{\partial \mathbf{x}} = \frac{\partial J(\mathbf{x})}{\partial \mathbf{x}} = \mathbf{0}. \quad (2.8)$$

In the case of a linear measurement model (where $\mathbf{h}(\mathbf{x}) = \mathbf{H}\mathbf{x}$), a closed form solution can be found to Eqn. 2.5. But in a more general case, where the measurement model is nonlinear, we need to resort to numerical methods, such as Newton-Raphson or Gauss-Newton.

2.1.3 Weighted Least Squares Estimation

The Method of Least Squares (LS) or WLS is a method that makes no probabilistic assumptions about the data. The only thing that is assumed is a signal model. The disadvantage of this is that no claims about optimality can be made and statistical performance cannot be assessed. However, the method is easy to implement and is, thus, widely used in practice.

The LS or WLS approach attempts only to minimize the squared difference between the given data $\mathbf{f} = (f_1, f_2, \dots, f_m)$ and the assumed signal model. The signal is deterministic and depends on the unknown vector of parameters $\mathbf{x}$. The LS estimator chooses the $\mathbf{x}$ that makes the signal model $\mathbf{h}(\mathbf{x})$ closest to the observed data $\mathbf{f}$. Closeness is measured by the LS error criterion:

$$J_{LS}(\mathbf{x}) = \sum_{i=1}^m (f_i - h_i(\mathbf{x}))^2 = (\mathbf{f} - \mathbf{h}(\mathbf{x}))^T (\mathbf{f} - \mathbf{h}(\mathbf{x})) \quad (2.9)$$

In the case where we have different levels of confidence in measurements, the WLS criterion is used:

$$J_{WLS}(\mathbf{x}) = \sum_{i=1}^m r_i^{-1} (f_i - h_i(\mathbf{x}))^2 = (\mathbf{f} - \mathbf{h}(\mathbf{x}))^T \mathbf{R}^{-1} (\mathbf{f} - \mathbf{h}(\mathbf{x})). \quad (2.10)$$

Note that if $\mathbf{f} - \mathbf{h}(\mathbf{x}) \sim \mathcal{N}(\mathbf{0}, \sigma^2 \mathbf{I})$, Eqn. 2.7 is practically identical to Eqn. 2.10 and, thus, the WLS estimator is also MLE. Obviously, the same solution methods apply for the minimization of Eqn. 2.10 (or Eqn. 2.9) as for the solution of Eqn. 2.8.

2.1.4 Least Absolute Value Estimation

Similarly to the LS (WLS) estimator, the Least Absolute Value (LAV) estimator makes no probabilistic assumptions about the data, and the only thing that is assumed is the signal model. The LAV approach attempts to minimize the absolute difference between the given data $\mathbf{f} = (f_1, f_2, \dots, f_m)$ and the assumed signal model. The signal is deterministic and depends on the unknown vector of parameters $\mathbf{x}$. The LAV estimator chooses the $\mathbf{x}$ that makes the signal model $\mathbf{h}(\mathbf{x})$ closest to the observed data $\mathbf{f}$. Closeness is measured by the LAV criterion:

$$J_{LAV}(\mathbf{x}) = \sum_{i=1}^m |f_i - h_i(\mathbf{x})|. \quad (2.11)$$

The value of the LAV estimator stems from its robustness in the face of outliers. The LAV estimator reduces the influence of bad data on the state estimation results and belongs to the class of robust estimators. In terms of solution, this problem can be recast in a linear programming problem that can be solved with any of the existing techniques, such as primal-dual interior-point method, Iteratively Re-weighted Least Squares (IRLS), or a simplex based algorithm. In section 3.4, an IRLS method for solving the LAV estimator is reviewed.

2.1.5 Schweppe-Huber's Generalized-M estimation based on projection statistics

The Schweppe-Huber's Generalized-M (SHGM) estimator is another estimator from the class of robust estimators. It implements a projection algorithm that accounts for the sparsity of the Jacobian matrix. It assigns to each data point a projection statistic. Based on these projection statistics, a robustly weighted estimator is defined. This estimator does not downweight good leverage points while bounding the influence of the bad ones [39].

The SHGM estimator attempts to minimize the following objective function:

$$J_{SHGM}(\mathbf{x}, \mathbf{a}) = \sum_{i=1}^m \gamma_i^2 \rho(r_i) \quad (2.12)$$

where the residual is defined as:

$$r_i = \frac{f_i - h_i(\mathbf{x})}{\gamma_i}, \quad (2.13)$$

and the ρ function is defined as:

$$\rho(r_i) = \begin{cases} \frac{1}{2} r_i^2 & |r_i| \leq c \\ c|r_i| - \frac{c^2}{2} & \text{otherwise} \end{cases} \quad (2.14)$$

where:

$$\gamma_i = \min \left\{ 1, \left(\frac{b_i}{PS_i} \right)^2 \right\}, \quad (2.15)$$

$$b_i = \chi_{\xi, 0.975}^2, \quad (2.16)$$

$$PS_i = \max_{\mathbf{H}_k} \frac{|\mathbf{H}_i \mathbf{H}_k^T|}{\beta_i} \quad k = 1, 2, \dots, m, \quad (2.17)$$

$$\beta_i = 1.1926 \cdot \text{lomed}_i \left\{ \text{lomed}_{i \neq j} |\mathbf{H}_i \mathbf{H}_k^T + \mathbf{H}_j \mathbf{H}_k^T| \right\}, \quad (2.18)$$

$\mathbf{H}_i$ is the i -th row vector of the Jacobian matrix $\mathbf{H}$, ξ is the number of nonzero entries in the corresponding (i -th) row of the Jacobian matrix, the lomed function is the low median of the given set, and a good choice for c ranges from 1 to 3 [39].

2.2 Network Model

In this section, a multiphase model of the distribution network will be developed. First, a unified three-phase branch model will be presented; then, model parameters for all the relevant network components, such as conductors, transformers, voltage regulators, and loads will be developed. Further, its use to build a complete network model will be demonstrated.

2.2.1 A Unified Three-Phase Branch Model

A unified, three-phase branch model is proposed here. The model can represent all three-phase distribution network elements. Conductors, transformers, tap changers, and voltage regulators in any connection, are all presented with the same model from which the branch-admittance matrices are derived. With the proposed model, voltage regulators or tap changers and conductor sections or transformers connected in series can be represented as a single branch (as is usually found in practice and also in IEEE distribution test feeders). In comparison with the existing approaches, we eliminate the requirement to introduce an additional bus (where a voltage regulator or a tap changer, and a conductor or a transformer are modeled as two separate branches), which increases the dimension of the problem. This is also beneficial from the computer-implementation point of view, since it simplifies the computer program's design.

Three-phase branch models that have been developed for the power-flow analysis of the distribution networks using a forward/backward sweep algorithm [40] are not applicable for this purpose in a straightforward manner, because matrix transformations are needed where singularities are encountered for several transformer models [41]. A modified version of the forward/backward sweep algorithm can be used for state estimation problems, but it lacks accuracy, as shown in [42]. References [43], [44] claim the use of three-phase component models with a WLS approach or its variation, but the relationships between the measurements and the state variables are not provided.

The proposed three-phase branch model is shown in Fig. 2.6. The model is symmetrical and establishes a relationship between the voltages and the currents on the nodes k and m . To simplify the formal derivation, the auxiliary p and q nodes are introduced. The proposed model is a three-phase extension of the single-phase, unified, branch model depicted in Fig. 2.7. that was developed by Monticelli in [45], and can be applied to networks where phase symmetry is assumed. Here, line-to-ground voltages are selected as the state variables

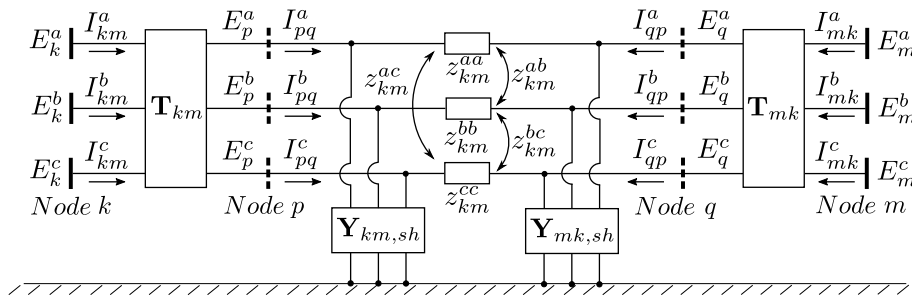


Figure 2.6: Three-phase unified branch model.

and, thus, the derivations of other variables depend on voltage phasors. The bus voltage phasors are denoted as:

$$E_m^a = |V_m^a| \cdot e^{j\phi_m^a} \quad (2.19)$$

where $|V_m^a|$ represents the phasor magnitude, and ϕ_m^a represents a phase angle at the node m on phase a . The elements $\mathbf{T}_{km}$ and $\mathbf{T}_{mk}$ are presented as matrices (Eqn. 2.20) and are

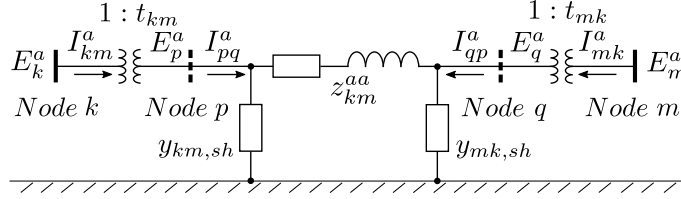


Figure 2.7: Single-phase unified branch model.

used for the modeling of voltage regulators or tap changers. Each phase voltage or current on the auxiliary node can be a linear combination of all three voltages or currents of the boundary node on their respective sides. This is necessary in order to correctly model the different regulator connections and tap settings.

$$\mathbf{T}_{km} = \begin{bmatrix} t_{km}^{aa} & t_{km}^{ab} & t_{km}^{ac} \\ t_{km}^{ba} & t_{km}^{bb} & t_{km}^{bc} \\ t_{km}^{ca} & t_{km}^{cb} & t_{km}^{cc} \end{bmatrix} \quad \mathbf{T}_{mk} = \begin{bmatrix} t_{mk}^{aa} & t_{mk}^{ab} & t_{mk}^{ac} \\ t_{mk}^{ba} & t_{mk}^{bb} & t_{mk}^{bc} \\ t_{mk}^{ca} & t_{mk}^{cb} & t_{mk}^{cc} \end{bmatrix} \quad (2.20)$$

The series impedance $\mathbf{Z}_{km}$ and the shunt admittance $\mathbf{Y}_{km,sh}$, $\mathbf{Y}_{mk,sh}$ matrices are sized as 3×3 elements:

$$\begin{aligned} \mathbf{Z}_{km} &= \begin{bmatrix} z_{km}^{aa} & z_{km}^{ab} & z_{km}^{ac} \\ z_{km}^{ba} & z_{km}^{bb} & z_{km}^{bc} \\ z_{km}^{ca} & z_{km}^{cb} & z_{km}^{cc} \end{bmatrix} = \mathbf{Y}_{km}^{-1} \\ \mathbf{Y}_{km,sh} &= \begin{bmatrix} y_{km,sh}^{aa} & y_{km,sh}^{ab} & y_{km,sh}^{ac} \\ y_{km,sh}^{ba} & y_{km,sh}^{bb} & y_{km,sh}^{bc} \\ y_{km,sh}^{ca} & y_{km,sh}^{cb} & y_{km,sh}^{cc} \end{bmatrix} \\ \mathbf{Y}_{mk,sh} &= \begin{bmatrix} y_{mk,sh}^{aa} & y_{mk,sh}^{ab} & y_{mk,sh}^{ac} \\ y_{mk,sh}^{ba} & y_{mk,sh}^{bb} & y_{mk,sh}^{bc} \\ y_{mk,sh}^{ca} & y_{mk,sh}^{cb} & y_{mk,sh}^{cc} \end{bmatrix} \end{aligned} \quad (2.21)$$

2.2.1.1 Current Equations

Since the line-to-ground bus voltages are selected as independent variables, the line currents are derived as functions of the line-to-ground bus voltages. The line currents can be a part of the measurement model in the formulation of the state-estimation problem, and are necessary in the derivation of line power flows. The line current for phase a with the support of auxiliary nodes can be written as:

$$I_{km}^a = I_{pq}^a \cdot t_{km}^{aa} + I_{pq}^b \cdot t_{km}^{ab} + I_{pq}^c \cdot t_{km}^{ac} = \sum_{j \in \Omega_p} I_{pq}^j \cdot t_{km}^{aj}, \quad (2.22)$$

where $\Omega_p = \{a, b, c\}$ denotes the phases. The current between the auxiliary buses can be expressed as:

$$\begin{aligned} I_{pq}^a &= (E_p^a - E_q^a) y_{km}^{aa} + (E_p^b - E_q^b) y_{km}^{ab} + (E_p^c - E_q^c) y_{km}^{ac} + E_p^a y_{km,sh}^{aa} + E_p^b y_{km,sh}^{ab} + E_p^c y_{km,sh}^{ac} \\ &= \sum_{n \in \Omega_p} ((E_p^n - E_q^n) y_{km}^{an} + E_p^n y_{km,sh}^{an}). \end{aligned} \quad (2.23)$$

The relationship between the voltage phasors at the auxiliary buses and the boundary buses is:

$$E_p^a = E_k^a t_{km}^{aa} + E_k^b t_{km}^{ab} + E_k^c t_{km}^{ac} = \sum_{i \in \Omega_p} E_k^i t_{km}^{ai} \quad (2.24)$$

$$E_q^a = E_m^a t_{mk}^{aa} + E_m^b t_{mk}^{ab} + E_m^c t_{mk}^{ac} = \sum_{i \in \Omega_p} E_m^i t_{mk}^{ai}. \quad (2.25)$$

Eqns. 2.24 and 2.25 are inserted into Eqn. 2.23:

$$I_{pq}^a = \sum_{n \in \Omega_p} \left[\left(\sum_{i \in \Omega_p} E_k^i t_{km}^{ni} - \sum_{i \in \Omega_p} E_m^i t_{mk}^{ni} \right) y_{km}^{an} + \sum_{i \in \Omega_p} E_k^i t_{km}^{ni} y_{km,sh}^{an} \right], \quad (2.26)$$

finally, Eqn. 2.26 is inserted into Eqn. 2.22:

$$I_{km}^a = \sum_{j \in \Omega_p} \sum_{n \in \Omega_p} \sum_{i \in \Omega_p} \left[(y_{km}^{jn} + y_{km,sh}^{jn}) t_{km}^{ni} t_{km}^{aj} E_k^i - t_{mk}^{ni} t_{km}^{aj} y_{km}^{jn} E_m^i \right]. \quad (2.27)$$

The derivation is limited to one phase, as others can be written analogously. Because of the model symmetry, the I_{mk}^a is simply:

$$I_{mk}^a = \sum_{j \in \Omega_p} \sum_{n \in \Omega_p} \sum_{i \in \Omega_p} \left[(y_{km}^{jn} + y_{mk,sh}^{jn}) t_{mk}^{ni} t_{mk}^{aj} E_m^i - t_{km}^{ni} t_{mk}^{aj} y_{km}^{jn} E_k^i \right]. \quad (2.28)$$

To obtain a branch-admittance matrix, a complete set of equations is written in matrix form in Eqn. 2.29, while Eqn. 2.30 displays the same set in a more compact form, which is also more convenient for computer implementation.

$$\begin{bmatrix} I_{km}^a \\ I_{km}^b \\ I_{km}^c \\ I_{mk}^a \\ I_{mk}^b \\ I_{mk}^c \end{bmatrix} = \begin{bmatrix} \sum_{j \in \Omega_p} \sum_{n \in \Omega_p} (y_{km}^{jn} + y_{km,sh}^{jn} t_{km}^{na} t_{km}^{aj}) & \cdots & - \sum_{j \in \Omega_p} \sum_{n \in \Omega_p} t_{mk}^{na} t_{km}^{aj} y_{km}^{jn} & \cdots \\ \sum_{j \in \Omega_p} \sum_{n \in \Omega_p} (y_{km}^{jn} + y_{km,sh}^{jn} t_{km}^{nb} t_{km}^{bj}) & \cdots & - \sum_{j \in \Omega_p} \sum_{n \in \Omega_p} t_{mk}^{nb} t_{km}^{bj} y_{km}^{jn} & \cdots \\ \vdots & \vdots & \vdots & \vdots \\ - \sum_{j \in \Omega_p} \sum_{n \in \Omega_p} t_{km}^{na} t_{mk}^{aj} y_{km}^{jn} & \cdots & \sum_{j \in \Omega_p} \sum_{n \in \Omega_p} (y_{km}^{jn} + y_{mk,sh}^{jn} t_{km}^{na} t_{km}^{aj}) & \cdots \\ - \sum_{j \in \Omega_p} \sum_{n \in \Omega_p} t_{km}^{nb} t_{mk}^{bj} y_{km}^{jn} & \cdots & \sum_{j \in \Omega_p} \sum_{n \in \Omega_p} (y_{km}^{jn} + y_{mk,sh}^{jn} t_{km}^{nb} t_{km}^{bj}) & \cdots \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} \cdot \begin{bmatrix} E_k^a \\ E_k^b \\ E_k^c \\ E_m^a \\ E_m^b \\ E_m^c \end{bmatrix} \quad (2.29)$$

$$\begin{bmatrix} \mathbf{I}_{km}^{\{a,b,c\}} \\ \mathbf{I}_{mk}^{\{a,b,c\}} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{T}_{km} \cdot (\mathbf{Y}_{km} + \mathbf{Y}_{km,sh}) \cdot \mathbf{T}_{km} & -\mathbf{T}_{km} \cdot \mathbf{Y}_{km} \cdot \mathbf{T}_{mk} \\ -\mathbf{T}_{mk} \cdot \mathbf{Y}_{km} \cdot \mathbf{T}_{km} & \mathbf{T}_{mk} \cdot (\mathbf{Y}_{km} + \mathbf{Y}_{mk,sh}) \cdot \mathbf{T}_{mk} \end{bmatrix}}_{\mathbf{Y}_{b,km}} \cdot \begin{bmatrix} \mathbf{E}_k^{\{a,b,c\}} \\ \mathbf{E}_m^{\{a,b,c\}} \end{bmatrix} \quad (2.30)$$

Another quantity that is often required by applications that use the model is line power flow. A relationship between the state variables and the active and reactive branch power flows is derived as follows. The conjugate of the complex power flow can be written as:

$$(S_{km}^a)^* = (E_k^a)^* \cdot I_{km}^a = P_{km}^a - jQ_{km}^a. \quad (2.31)$$

Considering Eqn. 2.19 and $y_{km}^{aa} = g_{km}^{aa} + jb_{km}^{aa}$, Eqn. 2.31 yields an expressions for the real (Eqn. 2.32) and reactive (Eqn. 2.33) power flows:

$$\begin{aligned} P_{km}^a = |V_k^a| \sum_{j \in \Omega_p} \sum_{n \in \Omega_p} \sum_{i \in \Omega_p} & \left\{ t_{km}^{ni} t_{km}^{aj} |V_k^i| \left[(g_{km}^{jn} + g_{km,sh}^{jn}) \cos(\phi_k^a - \phi_k^i) + (b_{km}^{jn} + b_{km,sh}^{jn}) \sin(\phi_k^a - \phi_k^i) \right] \right. \\ & \left. - t_{mk}^{ni} t_{km}^{aj} |V_m^i| \left[g_{km}^{jn} \cos(\phi_k^a - \phi_m^i) + b_{km}^{jn} \sin(\phi_k^a - \phi_m^i) \right] \right\}, \end{aligned} \quad (2.32)$$

$$Q_{km}^a = |V_k^a| \sum_{j \in \Omega_p} \sum_{n \in \Omega_p} \sum_{i \in \Omega_p} \left\{ t_{km}^{ni} t_{km}^{aj} |V_k^i| \left[(g_{km}^{jn} + g_{km,sh}^{jn}) \sin(\phi_k^a - \phi_k^i) - (b_{km}^{jn} + b_{km,sh}^{jn}) \cos(\phi_k^a - \phi_k^i) \right] \right. \\ \left. - t_{mk}^{ni} t_{km}^{aj} |V_m^i| \left[g_{km}^{jn} \sin(\phi_k^a - \phi_m^i) - b_{km}^{jn} \cos(\phi_k^a - \phi_m^i) \right] \right\}. \quad (2.33)$$

2.2.2 Conductors

The modeling of distribution overhead lines and underground cables is a critical step which must be taken before any analysis of the distribution feeder can take place. Here, the process of determining series impedance and shunt admittance matrices will be reviewed. Further, geometrical configurations of overhead lines and underground cables in IEEE reference distribution feeders will be reviewed. Series impedance and shunt admittance matrices for the unified branch model will be developed for those configurations.

2.2.2.1 Modified Carson's Equations

Carson's equations can be used to determine self and mutual impedances for an arbitrary number of overhead or underground conductors. Here, only the final equations will be reviewed, and derivation can be found in [40].

Self and mutual series impedance matrix elements are determined as:

$$\hat{z}_{ii-usa} = r_i + 0.0953016 + j0.1213422 \left(\ln \frac{1}{GMR_i} + 7.93401 \right) \Omega/\text{mile}, \quad (2.34)$$

$$\hat{z}_{ii-eu} = r_i + 0.049348 + j0.062832 \left(\ln \frac{1}{GMR_i} + 6.8371 \right) \Omega/\text{km}, \quad (2.35)$$

$$\hat{z}_{ij-usa} = 0.0953016 + j0.1213422 \left(\ln \frac{1}{GMR_i} + 7.93401 \right) \Omega/\text{mile}, \quad (2.36)$$

$$\hat{z}_{ij-eu} = 0.049348 + j0.062832 \left(\ln \frac{1}{D_{ij}} + 6.8371 \right) \Omega/\text{km}, \quad (2.37)$$

where:

- GMR_i ... Geometric Mean Radius of conductor i in feet in Eqn. 2.34, and in meters in Eqn. 2.35
- D_{ij} ... distance between conductors i and j in feet in Eqn. 2.36, and in meters in Eqn. 2.37

2.2.2.2 Series impedance matrices of overhead lines in IEEE distribution test feeders

The process of developing the series impedance matrix includes the following steps. First, conductor types and their geometric layout need to be determined. Then, primitive matrix elements are determined with the application of Carson's equations. Finally, the Kron reduction method is used to produce the final matrix that can be plugged into the unified branch model.

In IEEE distribution test feeders, there are three spacing models for overhead lines listed in Table 2.1. Figure 2.8 shows the geometrical configuration of the conductors for the spacings listed. Another necessary piece of information is the conductor data. Table B.1 lists the characteristics of the various conductors that are used for the overhead configurations in distribution feeders.

Table 2.1: Overhead Line Spacings.

Spacing ID	Type
500	Three-Phase, 4 Wire
505	Two-Phase, 3 Wire
510	Single-Phase, 2 Wire

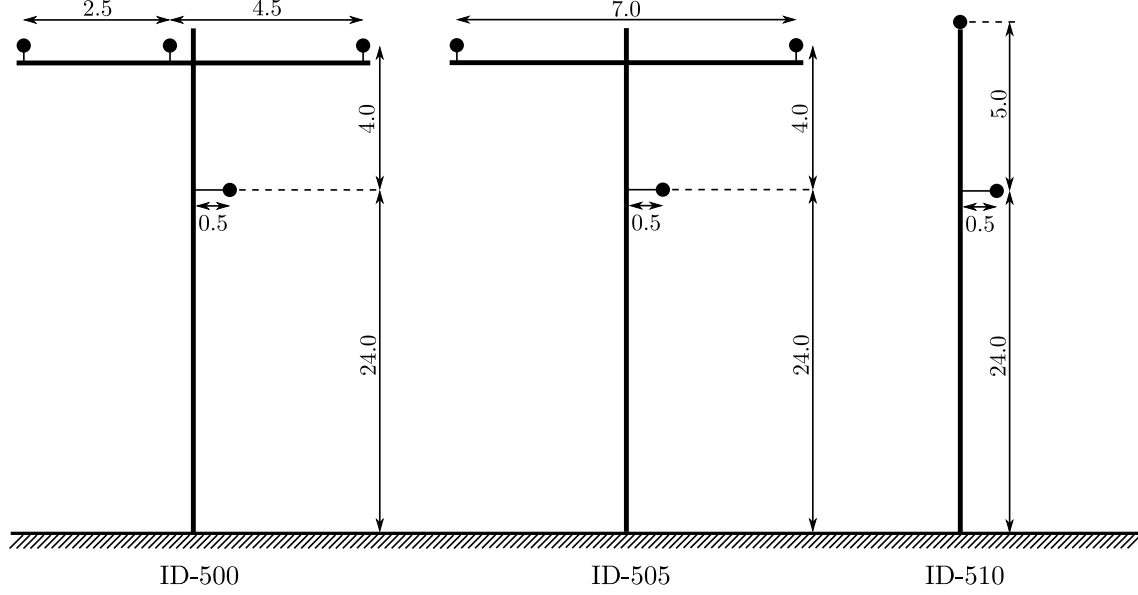


Figure 2.8: Overhead Line Spacings.

The topology of the IEEE 13-bus feeder is included in the Appendix B. Table A.7 lists the topology data. By matching this data with data in Table B.1, we extract the relevant GMR and resistance for each line segment. These two are then used in Eqn. 2.34 to determine the self impedance of the conductor section. Mutual impedances are determined by matching the geometrical configuration of the line segment with the configuration in Fig. 2.8, calculating the distance between the conductors and applying it to Eqn. 2.36. For example, the calculation of the primitive series impedance matrix for the line segment 632-633 is as follows:

$$\hat{\mathbf{z}}_{632-633} = \begin{bmatrix} \hat{z}_{aa} & \hat{z}_{ab} & \hat{z}_{ac} & \hat{z}_{an} \\ \hat{z}_{ba} & \hat{z}_{bb} & \hat{z}_{bc} & \hat{z}_{bn} \\ \hat{z}_{ca} & \hat{z}_{cb} & \hat{z}_{cc} & \hat{z}_{cn} \\ \hline \hat{z}_{na} & \hat{z}_{nb} & \hat{z}_{nc} & \hat{z}_{nn} \end{bmatrix} \quad (2.38)$$

$$= \begin{bmatrix} 0.6873 + 1.5465j & 0.0953 + 0.7802j & 0.0953 + 0.8515j & 0.0953 + 0.7865j \\ 0.0953 + 0.7802j & 0.6873 + 1.5465j & 0.0953 + 0.7266j & 0.0953 + 0.7674j \\ 0.0953 + 0.8515j & 0.0953 + 0.7266j & 0.6873 + 1.5465j & 0.0953 + 0.7524j \\ \hline 0.0953 + 0.7865j & 0.0953 + 0.7674j & 0.0953 + 0.7524j & 0.6873 + 1.5465j \end{bmatrix} \quad (2.39)$$

where the first three rows / columns represent phase conductors, and the fourth row / column represents the neutral conductor. The obtained matrix is reduced to a 3 by 3

matrix using the Kron reduction technique [40]:

$$\mathbf{z}_{abc} = \hat{\mathbf{z}}_{ij} - \hat{\mathbf{z}}_{in} \cdot \hat{\mathbf{z}}_{nn}^{-1} \cdot \hat{\mathbf{z}}_{nj} \quad (2.40)$$

where:

$$\hat{\mathbf{z}}_{ij} = \begin{bmatrix} 0.6873 + 1.5465j & 0.0953 + 0.7802j & 0.0953 + 0.8515j \\ 0.0953 + 0.7802j & 0.6873 + 1.5465j & 0.0953 + 0.7266j \\ 0.0953 + 0.8515j & 0.0953 + 0.7266j & 0.6873 + 1.5465j \end{bmatrix}, \quad (2.41)$$

$$\hat{\mathbf{z}}_{in} = \begin{bmatrix} 0.0953 + 0.7865j \\ 0.0953 + 0.7674j \\ 0.0953 + 0.7524j \end{bmatrix}, \quad (2.42)$$

$$\hat{\mathbf{z}}_{nj} = [0.0953 + 0.7865j \quad 0.0953 + 0.7674j \quad 0.0953 + 0.7524j], \quad (2.43)$$

$$\hat{\mathbf{z}}_{nn} = [0.6873 + 1.5465j]. \quad (2.44)$$

The series impedance matrix of branch 632-633 is thus:

$$\mathbf{z}_{632-633} = \begin{bmatrix} 0.7526 + 1.1814j & 0.1580 + 0.4237j & 0.1559 + 0.5017j \\ 0.1580 + 0.4237j & 0.7475 + 1.1983j & 0.1535 + 0.3850j \\ 0.1559 + 0.5017j & 0.1535 + 0.3850j & 0.7435 + 1.2113j \end{bmatrix}. \quad (2.45)$$

Line segments that have two-phase or single-phase configurations produce matrices where the missing conductors are zeroed out.

2.2.2.3 Shunt admittance matrices of overhead lines in IEEE distribution test feeders

The process of developing the shunt admittance matrix for the overhead line is as follows. First, self and mutual potential coefficients are determined from the following equations:

$$\hat{P}_{ii} = 11.17689 \cdot \ln \frac{S_{ii}}{RD_i} \text{mile}/\mu F, \quad (2.46)$$

$$\hat{P}_{ij} = 11.17689 \cdot \ln \frac{S_{ij}}{D_{ij}} \text{mile}/\mu F, \quad (2.47)$$

where:

- S_{ii} ... distance from conductor i to its image in the ground i'
- S_{ij} ... distance from conductor i to image in the ground of conductor j
- D_{ij} ... distance from conductor i to conductor j
- RD_i ... radius of conductor i

note that the values of S_{ii} , RD_i , D_{ij} , and S_{ij} must all be in the same units. Next, the primitive potential coefficient matrix is formed. For a four-wire grounded-wye line, the primitive coefficient matrix will be of the form:

$$\hat{\mathbf{P}}_{\text{primitive}} = \left[\begin{array}{ccc|c} \hat{P}_{aa} & \hat{P}_{ab} & \hat{P}_{ac} & \hat{P}_{an} \\ \hat{P}_{ba} & \hat{P}_{bb} & \hat{P}_{bc} & \hat{P}_{bn} \\ \hat{P}_{ca} & \hat{P}_{cb} & \hat{P}_{cc} & \hat{P}_{cn} \\ \hline \hat{P}_{na} & \hat{P}_{nb} & \hat{P}_{nc} & \hat{P}_{nn} \end{array} \right] = \left[\begin{array}{cc} \hat{\mathbf{P}}_{ij} & \hat{\mathbf{P}}_{in} \\ \hat{\mathbf{P}}_{nj} & \hat{\mathbf{P}}_{nn} \end{array} \right] \quad (2.48)$$

In the final step, since the neutral conductor is grounded, the Kron reduction method is used to produce a matrix of size *Number of phases* x *Number of phases*:

$$\mathbf{P}_{abc} = \hat{\mathbf{P}}_{ij} - \hat{\mathbf{P}}_{in} \cdot \hat{\mathbf{P}}_{nn}^{-1} \cdot \hat{\mathbf{P}}_{nj} \quad (2.49)$$

The inverse of the potential coefficient matrix will give the *Number of phases* x *Number of phases* capacitance matrix:

$$\mathbf{C}_{abc} = \mathbf{P}_{abc}^{-1}. \quad (2.50)$$

For a two-phase line configuration, the capacitance matrix Eqn. 2.50 will be 2 x 2 elements. Similarly, for a single-phase line configuration, the capacitance matrix will result in a single element. Normally, shunt conductance is neglected and, thus, the shunt admittance matrix is given by:

$$\mathbf{Y}_{km,sh} = j \cdot 2\pi f \cdot \mathbf{C}_{abc} \quad (2.51)$$

To insert the obtained shunt admittance matrix into the unified branch model, the matrix needs to be 3 x 3 elements. In the case of two-phase or single-phase line configurations, the row/column corresponding to the missing phase(s) needs to be zeroed out.

2.2.2.4 Series impedance matrices of underground cables in IEEE distribution test feeders

The process of developing the series impedance matrix for underground cables is very similar to the process for overhead lines described in Section 2.2.2.2. The differences are in geometric configurations of the cables and in the cables themselves.

In IEEE distribution test feeders, there are two cable spacings for underground cables. They are listed in Table 2.2 and depicted in Fig. 2.9.

Table 2.2: Underground Cable Spacings.

Spacing ID	Type
515	Three-Phase, 3 Cable
520	Two-Phase, 2 Cable

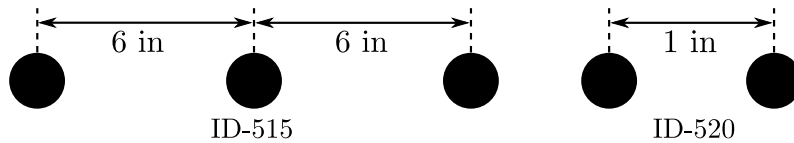


Figure 2.9: Underground Cable Spacings.

There are two popular types of underground cables, namely the concentric neutral cable, depicted in Fig. 2.10, and the tape-shielded cable, depicted in Fig. 2.11. In order to apply the modified Carson's equations, the resistance and GMR of the phase conductor and the equivalent neutral must be determined. To compute the resistance and GMR, the following data needs to be extracted from the datasheet of the underground cable:

- d_c ... phase conductor diameter in inches
- d_{od} ... nominal diameter over the concentric neutrals of the cable in inches
- d_s ... diameter of a concentric neutral strand in inches

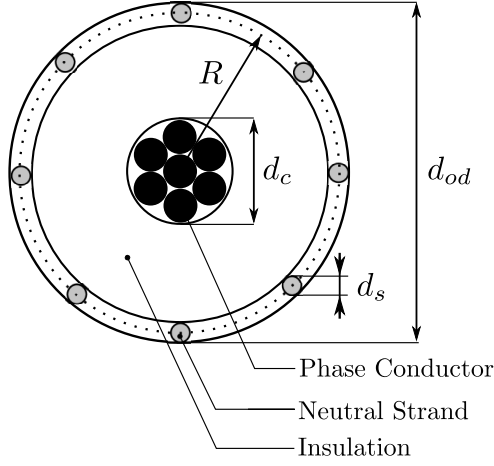


Figure 2.10: Concentric neutral cable.

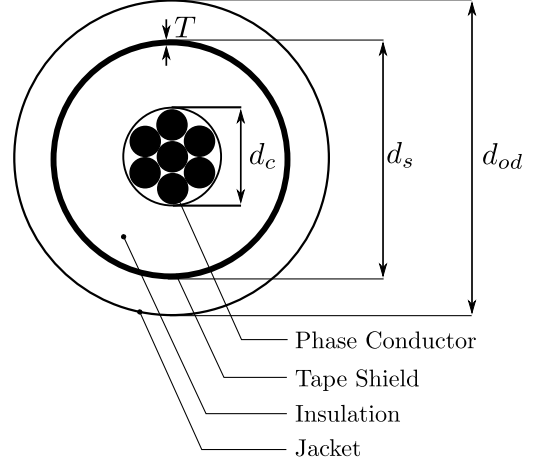


Figure 2.11: Tape-shielded cable.

- GMR_c ... geometric mean radius of the phase conductor in feet
- GMR_s ... geometric mean radius of a neutral strand in feet
- r_c ... resistance of the phase conductor in Ω/mile
- r_s ... resistance of a solid neutral strand in Ω/mile
- k ... number of concentric neutral strands

The equivalent geometric mean radius of the concentric neutral cable is calculated using:

$$GMR_{cn} = \sqrt[k]{GMR_s \cdot k \cdot R^{k-1}} \quad (2.52)$$

where R is the radius of a circle passing through the center of the concentric neutral strands and is determined as:

$$R = \frac{d_{od} - d_s}{24} \quad (2.53)$$

The equivalent resistance of the concentric neutral is determined using:

$$r_{cn} = \frac{r_s}{k} \quad (2.54)$$

The spacings between a concentric neutral and phase conductors and other concentric neutrals in the various configurations are as follows:

- Concentric neutral to its own phase conductor (see Fig. 2.10) $D_{ij} = R$
- Concentric neutral to an adjacent concentric neutral D_{ij} = center-to-center distance of the phase conductors.
- Concentric neutral to an adjacent phase conductor. The geometric mean distance between a concentric neutral and an adjacent phase conductor is given by:

$$D_{ij} = \sqrt[k]{D_{nm}^k - R^k} \quad (2.55)$$

where D_{nm} is the center-to-center distance between phase conductors. For configurations where the distance between cables is much greater than the radius R , the expression can be simplified into $D_{ij} = D_{nm}$.

In applying the modified Carson's equations, the numbering of phase and neutral conductors is important when constructing the primitive series impedance matrix. For example, a three-phase, three-cable configuration depicted in Fig. 2.9 must be numbered as:

- 1 is the phase conductor no. 1
- 2 is the phase conductor no. 2
- 3 is the phase conductor no. 3
- 4 is the neutral of conductor no. 1
- 5 is the neutral of conductor no. 2
- 6 is the neutral of conductor no. 3

the primitive series impedance matrix is thus:

$$\hat{\mathbf{z}}_{\text{primitive}} = \begin{bmatrix} \hat{z}_{11} & \hat{z}_{12} & \hat{z}_{13} & \hat{z}_{14} & \hat{z}_{15} & \hat{z}_{16} \\ \hat{z}_{21} & \hat{z}_{22} & \hat{z}_{23} & \hat{z}_{24} & \hat{z}_{25} & \hat{z}_{26} \\ \hat{z}_{31} & \hat{z}_{32} & \hat{z}_{33} & \hat{z}_{34} & \hat{z}_{35} & \hat{z}_{36} \\ \hat{z}_{41} & \hat{z}_{42} & \hat{z}_{43} & \hat{z}_{44} & \hat{z}_{45} & \hat{z}_{46} \\ \hat{z}_{51} & \hat{z}_{52} & \hat{z}_{53} & \hat{z}_{54} & \hat{z}_{55} & \hat{z}_{56} \\ \hat{z}_{61} & \hat{z}_{62} & \hat{z}_{63} & \hat{z}_{64} & \hat{z}_{65} & \hat{z}_{66} \end{bmatrix} = \left[\begin{array}{c|c} \hat{\mathbf{z}}_{ij} & \hat{\mathbf{z}}_{in} \\ \hline \hat{\mathbf{z}}_{nj} & \hat{\mathbf{z}}_{nn} \end{array} \right] \quad (2.56)$$

with the application of the Kron reduction method (Eqn. 2.40), the *number of phases* x *number of phases* series impedance matrix is obtained.

Table B.2 lists the characteristics of the various concentric neutral underground cables that are present in IEEE distribution test feeders.

The tape-shielded cables consist of a central phase conductor covered by a thin layer of nonmetallic, semi-conducting screen to which the insulating material is bonded. The insulation is covered by a semiconducting insulation screen. The bare copper shield is helically applied around the insulation screen. An insulating jacket surrounds the tape shield. The relevant geometric parameters are:

- d_c ... diameter of the phase conductor in inches
- d_s ... outside diameter of the tape shield in inches
- d_{od} ... outside diameter over the jacket in inches
- T ... thickness of the copper tape shield in mils

To calculate the elements of the primitive series impedance matrix, modified Carson's equations are applied. The resistance and GMR of the phase conductor are found in Table B.1. The resistance of the tape shield is given by:

$$r_{\text{shield}} = 7.9385 \cdot 10^8 \frac{\rho}{d_s \cdot T} \Omega/\text{mile} \quad (2.57)$$

The resistivity (ρ) must be expressed in Ω -meters at 50°C . The outside diameter of the tape shield (d_s) must be in inches and the thickness of the tape shield (T) in mils.

The GMR of the tape shield is the radius of a circle passing through the middle of the shield and is given by:

$$GMR_{\text{shield}} = \frac{d_s - \frac{T}{1000}}{24}. \quad (2.58)$$

The spacings between the tape shields and conductors in various configurations are as follows:

- Tape shield to its own phase conductor: $D_{ij} = GMR_{\text{shield}} \dots$ radius to midpoint of the shield in feet
- Tape shield to an adjacent tape shield $D_{ij} =$ center-to-center distance of the phase conductors in feet
- Tape shield to an adjacent phase or neutral conductor $D_{ij} =$ center-to-center distance between phase conductors in feet

again, the Kron method is applied to reduce the primitive series impedance matrix into a *number of phases* x *number of phases* matrix.

2.2.2.5 Shunt admittance matrices of underground cables in IEEE distribution test feeders

Most underground distribution lines consist of one or more tape-shielded or concentric neutral cables. The process of developing the shunt admittance matrix for any of the two configurations is the same as the one described in Section 2.2.2.3. To form a primitive admittance matrix, self and mutual potential coefficients need to be determined from Eqns. 2.46 and 2.47.

In the concentric neutral cable, all neutral strands are at ground potential. Neutral stranding acts as a Faraday cage, and confines the electric field created by the phase conductor within the boundary of the concentric neutral strands. This enables us to use relative permittivity of the insulation material as the relative permittivity of the medium ϵ_r . Capacitance from phase to ground for this type of cable is given by:

$$C_{ph} = \frac{2\pi\epsilon}{\ln \frac{R}{RD_c} - \frac{1}{k} \ln \frac{k \cdot RD_s}{R}} \quad (2.59)$$

where (see Fig. 2.10 for reference):

- $\epsilon = \epsilon_0 \epsilon_r \dots$ permittivity of the medium
- $\epsilon_r \dots$ relative permittivity of the medium
- $\epsilon_0 \dots$ permittivity of free space
- $RD_c = \frac{d_c}{2}$
- $RD_s = \frac{d_s}{2}$

From Eqn. 2.59, cable shunt admittance is obtained using:

$$\mathbf{Y}_{\text{shunt}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot j \cdot 2\pi f \cdot C_{ph} \quad (2.60)$$

In the tape-shielded cable, as with the concentric neutral, the electric field is confined to the insulation. The capacitance of the tape-shielded cable can be derived from the expression developed for the concentric neutral cable (Eqn. 2.59). The tape shield can be thought of as an infinite number of neutral strands:

$$C_{ag} = \lim_{k \rightarrow \infty} C_{ph} = \frac{2\pi\epsilon}{\ln \frac{R}{RD_c}}. \quad (2.61)$$

As with the concentric neutral cable, the cable shunt admittance matrix is obtained as:

$$\mathbf{Y}_{\text{shunt}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot j \cdot 2\pi f \cdot C_{ag} \quad (2.62)$$

2.2.3 Transformers

In modeling the transformers, we consider that distribution three-phase transformers are mostly wound on a common core and, thus, all windings are coupled to all other windings. A two-windings three-phase transformer then has a primitive or unconnected network consisting of six coupled coils. This type of transformer is depicted in Fig. 2.12.

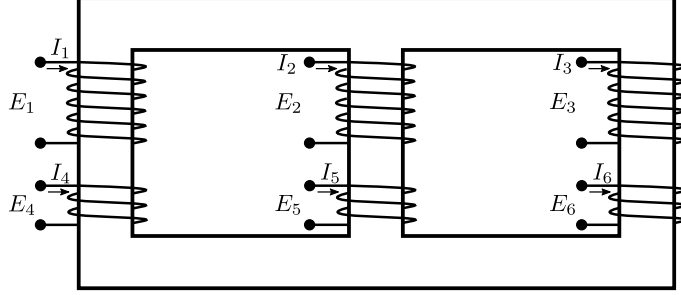


Figure 2.12: Two-winding three-phase transformer.

From this basic representation, the primitive network can be derived shown in Fig. 2.13. The dotted lines represent parasitic coupling between phases. This network is described by the primitive admittance matrix:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} & y_{13} & y_{14} & y_{15} & y_{16} \\ y_{21} & y_{22} & y_{23} & y_{24} & y_{25} & y_{26} \\ y_{31} & y_{32} & y_{33} & y_{34} & y_{35} & y_{36} \\ y_{41} & y_{42} & y_{43} & y_{44} & y_{45} & y_{46} \\ y_{51} & y_{52} & y_{53} & y_{54} & y_{55} & y_{56} \\ y_{61} & y_{62} & y_{63} & y_{64} & y_{65} & y_{66} \end{bmatrix} \cdot \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \\ E_5 \\ E_6 \end{bmatrix} \quad (2.63)$$

Considering the reciprocal nature of the mutual couplings and assuming that the flux

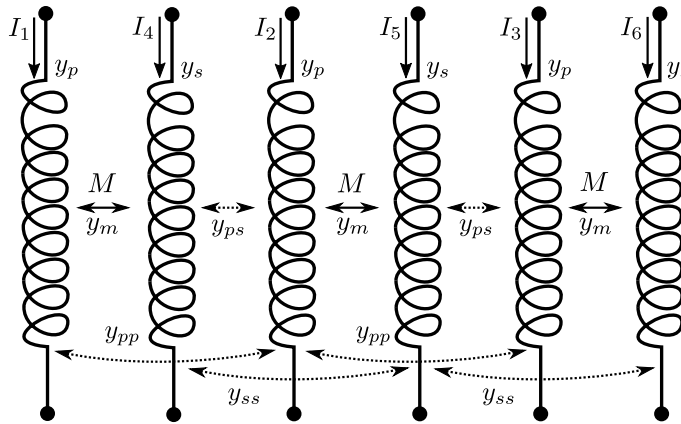


Figure 2.13: Two-winding three-phase transformer.

densities are symmetrically distributed between all windings, Eqn. 2.63 can be simplified

to:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} y_p & y_{pp} & y_{pp} & -y_m & y_{ps} & y_{ps} \\ y_{pp} & y_p & y_{pp} & y_{ps} & -y_m & y_{ps} \\ y_{pp} & y_{pp} & y_p & y_{ps} & y_{ps} & -y_m \\ -y_m & y_{ps} & y_{ps} & y_s & y_{ss} & y_{ss} \\ y_{ps} & -y_m & y_{ps} & y_{ss} & y_s & y_{ss} \\ y_{ps} & y_{ps} & -y_m & y_{ss} & y_{ss} & y_s \end{bmatrix} \cdot \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \\ E_5 \\ E_6 \end{bmatrix} \quad (2.64)$$

where:

- y_p is the admittance of the primary coil
- y_s is the admittance of the secondary coil
- y_m is the mutual admittance between primary and secondary coils on the same core
- y_{pp} is the mutual admittance between primary coils
- y_{ps} is the mutual admittance between primary and secondary coils on different cores
- y_{ss} is the mutual admittance between secondary coils

For three separate single-phase units, the y_{pp} , y_{ps} , and y_{ss} are zero. In three-phase units, those values represent interphase coupling and can have a noticeable effect. However, for simplicity reasons, these effects will be ignored in our analysis, and three-phase units will be modeled as connected single-phase units. The simplifications produce the following model:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} y_p & 0 & 0 & M & 0 & 0 \\ 0 & y_p & 0 & 0 & M & 0 \\ 0 & 0 & y_p & 0 & 0 & M \\ M & 0 & 0 & y_s & 0 & 0 \\ 0 & M & 0 & 0 & y_s & 0 \\ 0 & 0 & M & 0 & 0 & y_s \end{bmatrix} \cdot \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \\ E_5 \\ E_6 \end{bmatrix} = \mathbf{Y}_{\text{prim}} \cdot \mathbf{E}_{\text{prim}}. \quad (2.65)$$

The network admittance matrix for any two-winding three-phase transformer can be formed by the method of linear transformation. As an example, the formation of the admittance matrix for a grounded wye-delta connection is considered. The transformer connection is illustrated in Fig. 2.14, and the connection matrix $\mathbf{C}$ relating the branch voltages to the node voltages is:

$$\begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \\ E_5 \\ E_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} E_k^a \\ E_k^b \\ E_k^c \\ E_m^a \\ E_m^b \\ E_m^c \end{bmatrix} = \mathbf{C} \cdot \mathbf{E}_{\text{node}} \quad (2.66)$$

The nodal admittance matrix $\mathbf{Y}_{\text{node}}$ is calculated by:

$$\mathbf{Y}_{\text{node}} = \mathbf{C}^T \cdot \mathbf{Y}_{\text{prim}} \cdot \mathbf{C}. \quad (2.67)$$

If the primitive admittances are expressed in per unit, then the considered grounded wye-delta transformer must include an effective turns ratio $\sqrt{3}$. The upper right and lower

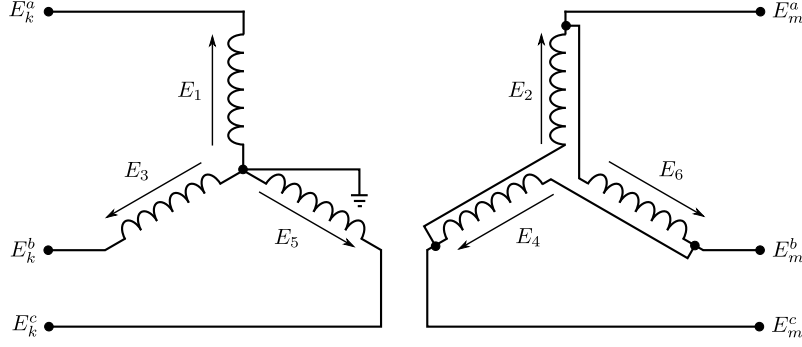


Figure 2.14: The grounded wye-delta connection.

left quadrants of matrix $\mathbf{Y}_{\text{node}}$ must be divided by $\sqrt{3}$ and the lower right quadrant by 3. Considering this, the nodal admittance model becomes:

$$\begin{bmatrix} I_k^a \\ I_k^b \\ I_k^c \\ I_m^a \\ I_m^b \\ I_m^c \end{bmatrix} = \begin{bmatrix} y & 0 & 0 & -y/\sqrt{3} & -y/\sqrt{3} & 0 \\ 0 & y & 0 & 0 & -y/\sqrt{3} & y/\sqrt{3} \\ 0 & 0 & y & y/\sqrt{3} & 0 & -y/\sqrt{3} \\ -y/\sqrt{3} & 0 & y/\sqrt{3} & \frac{2}{3}y & -\frac{1}{3}y & -\frac{1}{3}y \\ y/\sqrt{3} & -y/\sqrt{3} & 0 & -\frac{1}{3}y & \frac{2}{3}y & -\frac{1}{3}y \\ 0 & y/\sqrt{3} & -y/\sqrt{3} & -\frac{1}{3}y & -\frac{1}{3}y & \frac{2}{3}y \end{bmatrix} \cdot \begin{bmatrix} E_k^a \\ E_k^b \\ E_k^c \\ E_m^a \\ E_m^b \\ E_m^c \end{bmatrix}. \quad (2.68)$$

The models for the other common connections can be derived using the same method. In general, the transformer branch model can be represented by:

$$\begin{bmatrix} \mathbf{I}_k^{\{a,b,c\}} \\ \mathbf{I}_m^{\{a,b,c\}} \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{pp} & \mathbf{Y}_{ps} \\ \mathbf{Y}_{sp} & \mathbf{Y}_{ss} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{E}_k^{\{a,b,c\}} \\ \mathbf{E}_m^{\{a,b,c\}} \end{bmatrix}, \quad (2.69)$$

and it should be noted that the following relationship holds:

$$\mathbf{Y}_{sp} = \mathbf{Y}_{ps}^T. \quad (2.70)$$

In our model, the parameters of all three phases are assumed balanced. The most common three-phase connection can be modeled by three basic submatrices:

$$\mathbf{Y}_I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} y, \quad (2.71)$$

$$\mathbf{Y}_{II} = \frac{1}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} y, \quad (2.72)$$

$$\mathbf{Y}_{III} = \frac{1}{\sqrt{3}} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \end{bmatrix} y, \quad (2.73)$$

where y is the specified transformer leakage admittance. Table 2.3 and Table 2.4 list the combinations of $\mathbf{Y}_I$, $\mathbf{Y}_{II}$, and $\mathbf{Y}_{III}$ for the most common step-up and step-down transformer connections, respectively. It must be noted that off-nominal tap ratios are not considered in the node admittance matrix. They must be modeled separately in the branch as $\mathbf{T}_{km}$ and $\mathbf{T}_{mk}$ matrices.

Table 2.3: Submatrices for common step-down transformer connections.

Primary	Secondary	$\mathbf{Y}_{pp}$	$\mathbf{Y}_{ss}$	$\mathbf{Y}_{ps}$	$\mathbf{Y}_{sp}$
Y_g	Y_g	$\mathbf{Y}_I$	$\mathbf{Y}_I$	$-\mathbf{Y}_I$	$-\mathbf{Y}_I$
Y_g	Y	$\mathbf{Y}_{II}$	$\mathbf{Y}_{II}$	$-\mathbf{Y}_{II}$	$-\mathbf{Y}_{II}$
Y_g	Δ	$\mathbf{Y}_I$	$\mathbf{Y}_{II}$	$\mathbf{Y}_{III}$	$\mathbf{Y}_{III}^T$
Y	Y_g	$\mathbf{Y}_{II}$	$\mathbf{Y}_{II}$	$-\mathbf{Y}_{II}$	$-\mathbf{Y}_{II}$
Y	Y	$\mathbf{Y}_{II}$	$\mathbf{Y}_{II}$	$-\mathbf{Y}_{II}$	$-\mathbf{Y}_{II}$
Y	Δ	$\mathbf{Y}_{II}$	$\mathbf{Y}_{II}$	$\mathbf{Y}_{III}$	$\mathbf{Y}_{III}^T$
Δ	Y_g	$\mathbf{Y}_{II}$	$\mathbf{Y}_I$	$\mathbf{Y}_{III}$	$\mathbf{Y}_{III}^T$
Δ	Y	$\mathbf{Y}_{II}$	$\mathbf{Y}_{II}$	$\mathbf{Y}_{III}$	$\mathbf{Y}_{III}^T$
Δ	Δ	$\mathbf{Y}_{II}$	$\mathbf{Y}_{II}$	$-\mathbf{Y}_{II}$	$-\mathbf{Y}_{II}$

Table 2.4: Submatrices for common step-up transformer connections.

Primary	Secondary	$\mathbf{Y}_{pp}$	$\mathbf{Y}_{ss}$	$\mathbf{Y}_{ps}$	$\mathbf{Y}_{sp}$
Y_g	Y_g	$\mathbf{Y}_I$	$\mathbf{Y}_I$	$-\mathbf{Y}_I$	$-\mathbf{Y}_I$
Y_g	Y	$\mathbf{Y}_{II}$	$\mathbf{Y}_{II}$	$-\mathbf{Y}_{II}$	$-\mathbf{Y}_{II}$
Y_g	Δ	$\mathbf{Y}_I$	$\mathbf{Y}_{II}$	$\mathbf{Y}_{III}^T$	$\mathbf{Y}_{III}$
Y	Y_g	$\mathbf{Y}_{II}$	$\mathbf{Y}_{II}$	$-\mathbf{Y}_{II}$	$-\mathbf{Y}_{II}$
Y	Y	$\mathbf{Y}_{II}$	$\mathbf{Y}_{II}$	$-\mathbf{Y}_{II}$	$-\mathbf{Y}_{II}$
Y	Δ	$\mathbf{Y}_{II}$	$\mathbf{Y}_{II}$	$\mathbf{Y}_{III}^T$	$\mathbf{Y}_{III}$
Δ	Y_g	$\mathbf{Y}_{II}$	$\mathbf{Y}_I$	$\mathbf{Y}_{III}^T$	$\mathbf{Y}_{III}$
Δ	Y	$\mathbf{Y}_{II}$	$\mathbf{Y}_{II}$	$\mathbf{Y}_{III}^T$	$\mathbf{Y}_{III}$
Δ	Δ	$\mathbf{Y}_{II}$	$\mathbf{Y}_{II}$	$-\mathbf{Y}_{II}$	$-\mathbf{Y}_{II}$

2.2.4 Voltage Regulators

As the loads on the distribution feeders vary, so do the feeder voltages. Voltage regulators are installed in distribution systems in order to control the voltage so that every customer's voltage remains within an acceptable level. The inner workings of the regulator and different methods of compensation are widely discussed in [40] and [46]. In this subsection, only the relation between the input and output quantities is of interest. The fundamental concepts of a single-phase voltage regulator will be reviewed in subsection 2.2.4.1 in order to derive branch admittance matrices for various three-phase connections in subsection 2.2.4.2

2.2.4.1 Fundamental Concepts

Voltage regulator models can be derived from autotransformer models. An autotransformer is constructed by utilizing a two-winding transformer and connecting the two windings in series. The two possible connections, i.e. "buck" and "boost", are depicted in Fig. 2.15 and Fig. 2.16, respectively [46]. The two single-phase voltage regulators derived from these two possible connections are depicted in Fig. 2.17 and Fig. 2.18. The depicted situations represent the extreme "buck" and "boost" positions on a standard single-phase regulator. The resulting winding ratios for these configurations are $\frac{N_2 \pm N_1}{N_1}$. The basic equation relating the primary and secondary voltage sides is:

$$\frac{E_P}{E_S} = \frac{E_A - E_b}{E_a - E_b} = a_{ab}. \quad (2.74)$$

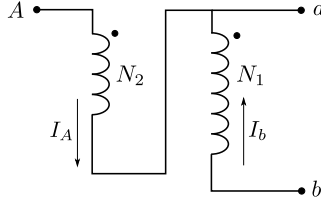


Figure 2.15: Bucking autotransformer configuration.

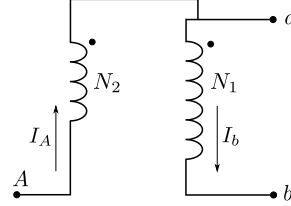


Figure 2.16: Boosting autotransformer configuration.

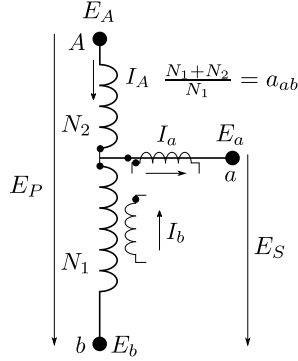


Figure 2.17: Bucking single-phase voltage regulator.

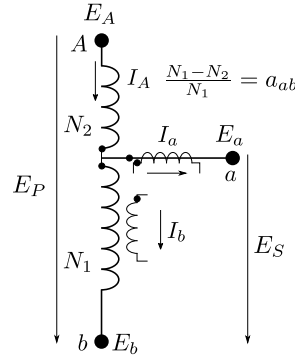


Figure 2.18: Boosting single-phase voltage regulator.

From this equation, the $\mathbf{T}_{km}$ and $\mathbf{T}_{mk}$ matrices of the unified branch model can be derived for any possible three-phase connection of the voltage regulator.

2.2.4.2 Three-Phase Connections

A bank of single-phase regulators connected in the grounded-wye configuration is shown in Fig. 2.19. Each of the three regulators controls the voltage delivered to the customers on each phase. The regulators act independently based on the loading and impedance of an individual feeder phase. Each regulator can regulate voltage in its full range. The standard single-phase voltage regulator has 32 steps of regulation and a $\pm 10\%$ range. Steps are symmetrically distributed around zero, therefore, for a total range of $\pm 10\%$, 32 steps yield a $\frac{5}{8}\%$ step size.

If we consider Eqn. 2.74, the relationship between primary and secondary side for wye-connected single-phase regulator is:

$$\begin{bmatrix} E_A \\ E_B \\ E_C \end{bmatrix} = \begin{bmatrix} a_a & 0 & 0 \\ 0 & a_b & 0 \\ 0 & 0 & a_c \end{bmatrix} \cdot \begin{bmatrix} E_a \\ E_b \\ E_c \end{bmatrix} = \mathbf{T} \cdot \mathbf{E}^{\{a,b,c\}}. \quad (2.75)$$

The line-to-neutral voltages are just scaled with the appropriate regulator ratio.

A bank of delta-connected regulators in boost configuration is depicted in Fig. 2.20. The voltage that is applied to the control is proportional to the line-to-line voltage across the load. Again, considering Eqn. 2.74, the relationship between line-to-neutral primary and secondary voltages is:

$$\begin{bmatrix} E_A \\ E_B \\ E_C \end{bmatrix} = \begin{bmatrix} a_{ab} & (1 - a_{ab}) & 0 \\ 0 & a_{bc} & (1 - a_{bc}) \\ (1 - a_{ca}) & 0 & a_{ca} \end{bmatrix} \cdot \begin{bmatrix} E_a \\ E_b \\ E_c \end{bmatrix} = \mathbf{T} \cdot \mathbf{E}^{\{a,b,c\}}. \quad (2.76)$$

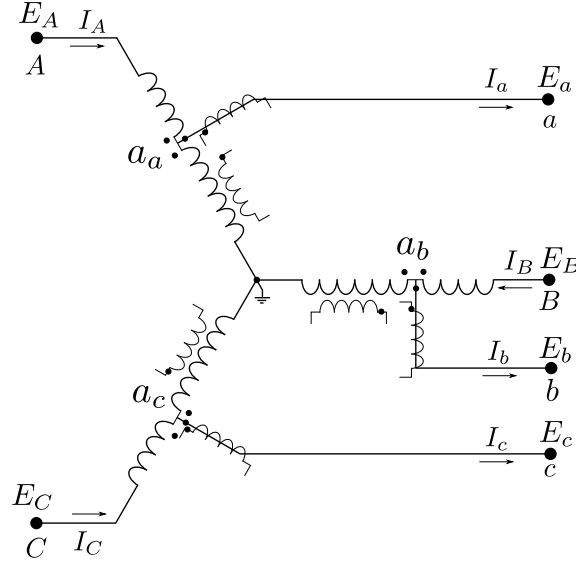


Figure 2.19: Single-phase regulators connected in grounded-wye.

Fig. 2.21 depicts primary and secondary line-to-line and line-to-neutral phasors for the

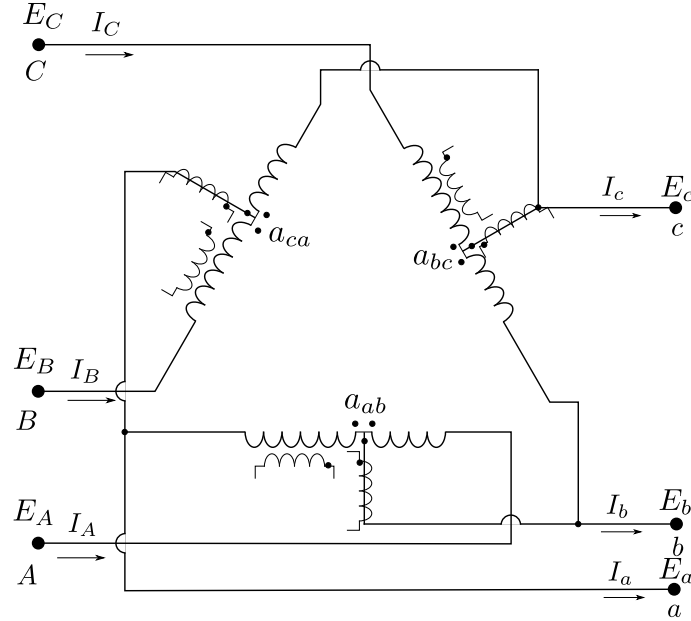


Figure 2.20: Single-phase regulators in a closed delta connection.

closed delta connected voltage regulator with the following tap ratios: $a_{ca} = 0.93$, $a_{bc} = 0.9$, and $a_{ab} = 1.1$. It is obvious that besides magnitude scaling, phase shifts also occur. It should also be noted that the range of regulation in terms of line-to-neutral voltages is less than $\pm 10\%$ (the range of one single-phase voltage regulator unit).

A bank of open delta connected single-phase voltage regulators is depicted in Fig. 2.22. Considering Eqn. 2.74, the relationships between primary and secondary voltages

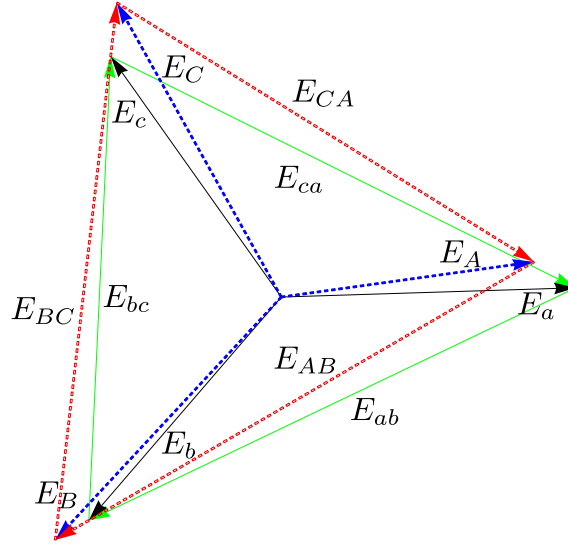


Figure 2.21: Phase diagram for closed delta connection.

are derived:

$$\begin{bmatrix} E_A \\ E_B \\ E_C \end{bmatrix} = \begin{bmatrix} a_{ab} & (1 - a_{ab}) & 0 \\ 0 & 1 & 0 \\ 0 & (1 - a_{cb}) & a_{cb} \end{bmatrix} \cdot \begin{bmatrix} E_a \\ E_b \\ E_c \end{bmatrix} = \mathbf{T} \cdot \mathbf{E}^{\{a,b,c\}}. \quad (2.77)$$

Fig. 2.23 depicts primary and secondary line-to-line and line-to-neutral phasors for the open-delta connected voltage regulator.

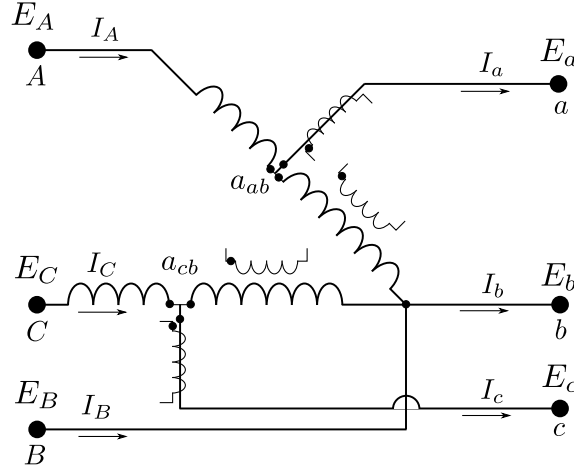


Figure 2.22: Two single-phase regulators in an open delta connection.

2.2.5 The complete network model

To construct a complete network model, a system-admittance matrix is needed. With branch matrices for possible components developed, the construction of a system-admittance matrix is a straightforward process. The proposed unified branch model is applied to a fictitious 4-bus system shown in Fig. 2.24. The example includes a transformer in a delta-grounded wye connection, a conductor section with a voltage regulator in an open-delta connection, and an additional conductor section. The following example covers the use of

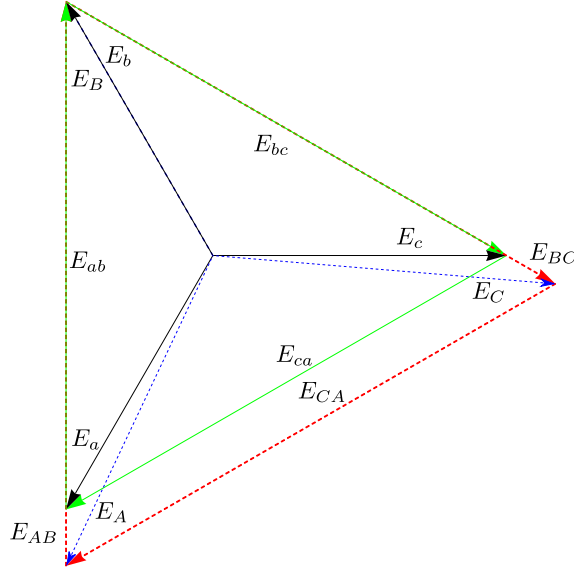


Figure 2.23: Phase diagram for an open delta connection.

the unified branch model for the majority of practical scenarios. Other possible cases can be inferred from the demonstrated ones. The three example branches have the following branch matrices:

- *The 1-2 branch:* The transformer leakage-admittance matrix is constructed from the basic sub-matrices in Table 2.4. For the transformer in the example, the sub-matrices are: $\mathbf{Y}_{pp} = \mathbf{Y}_{II}$, $\mathbf{Y}_{pq} = \mathbf{Y}_{III}$, $\mathbf{Y}_{qp} = \mathbf{Y}_{III}^T$, and $\mathbf{Y}_{qq} = \mathbf{Y}_I$. Possible off-nominal tap ratios are considered in the $\mathbf{T}_{12}$ and $\mathbf{T}_{21}$ matrices. The complete branch-admittance matrix is:

$$\mathbf{Y}_{b,12} = \begin{bmatrix} \mathbf{T}_{12} \cdot (\mathbf{Y}_{pp} + \mathbf{Y}_{km,sh}) \cdot \mathbf{T}_{12} & -\mathbf{T}_{12} \cdot \mathbf{Y}_{pq} \cdot \mathbf{T}_{21} \\ -\mathbf{T}_{21} \cdot \mathbf{Y}_{qp} \cdot \mathbf{T}_{12} & \mathbf{T}_{21} \cdot (\mathbf{Y}_{qq} + \mathbf{Y}_{mk,sh}) \cdot \mathbf{T}_{21} \end{bmatrix} \quad (2.78)$$

- *The 2-3 branch:* The branch consists of a conductor section and a voltage regulator at the side of bus 3. The resulting branch matrix is:

$$\mathbf{Y}_{b,23} = \begin{bmatrix} (\mathbf{Y}_{23} + \mathbf{Y}_{23,sh}) & \mathbf{Y}_{23} \cdot \mathbf{T}_{32} \\ -\mathbf{T}_{32} \cdot \mathbf{Y}_{23} & \mathbf{T}_{32} \cdot (\mathbf{Y}_{23} + \mathbf{Y}_{23,sh}) \cdot \mathbf{T}_{32} \end{bmatrix} \quad (2.79)$$

- *The 3-4 branch:* The branch consists of a conductor section only. The resulting branch matrix is:

$$\mathbf{Y}_{b,34} = \begin{bmatrix} (\mathbf{Y}_{34} + \mathbf{Y}_{34,sh}) & -\mathbf{Y}_{34} \\ -\mathbf{Y}_{34} & (\mathbf{Y}_{34} + \mathbf{Y}_{43,sh}) \end{bmatrix} \quad (2.80)$$

The system-admittance matrix is constructed from the developed branch matrices. Branch sub-matrices are added to the corresponding elements in the system matrix. The final result (superscripts indicate the inner sub-matrix of every branch matrix) is:

$$\mathbf{Y}_{sys} = \begin{bmatrix} \mathbf{Y}_{b,12}^{11} & \mathbf{Y}_{b,12}^{12} & & \\ \mathbf{Y}_{b,12}^{21} & \mathbf{Y}_{b,12}^{22} + \mathbf{Y}_{b,23}^{11} & \mathbf{Y}_{b,23}^{12} & \\ & \mathbf{Y}_{b,23}^{21} & \mathbf{Y}_{b,23}^{22} + \mathbf{Y}_{b,34}^{11} & \mathbf{Y}_{b,34}^{12} \\ & & \mathbf{Y}_{b,34}^{21} & \mathbf{Y}_{b,34}^{22} \end{bmatrix}. \quad (2.81)$$

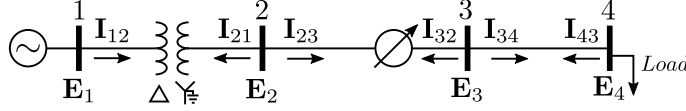


Figure 2.24: Example system.

With the system-admittance matrix available, Eqn. 2.82 and Eqn. 2.83 show expressions for the bus power injections. G_{km}^{ap} and B_{km}^{ap} , respectively, represent the real and imaginary components of the (a, p) and (k, m) system-admittance submatrix, and Λ is the set of buses adjacent to bus k , including bus k .

$$P_k^a = |V_k^a| \sum_{m \in \Lambda} \sum_{p \in \Omega_p} |V_m^p| (G_{km}^{ap} \cos(\phi_k^a - \phi_m^p) + B_{km}^{ap} \sin(\phi_k^a - \phi_m^p)) \quad (2.82)$$

$$Q_k^a = |V_k^a| \sum_{m \in \Lambda} \sum_{p \in \Omega_p} |V_m^p| (G_{km}^{ap} \sin(\phi_k^a - \phi_m^p) - B_{km}^{ap} \cos(\phi_k^a - \phi_m^p)) \quad (2.83)$$

2.2.6 Summary

This chapter presented what SE is in the context of signal processing, and how it relates to power systems. As SE is already universally adopted in power transmission networks and is making its way into power distribution networks, the main differences between the two networks were described, and the reasons preventing a straightforward application of SE were highlighted.

Further, different types of estimators were reviewed, namely MLE, WLS, LAV, and SHGM, based on projection statistics. It was shown that WLS is identical to MLE if we assume normal noise distribution in measurements. Besides their statistical efficiency, which is a property that tells us how much information can be obtained from the data using a specific estimator, estimators were also classified by their robustness, which is a property that tells us how much bad data can be present in the input measurement set that the estimator will implicitly reject.

Models for three-phase underground cables, overhead lines, transformers, tap changers, and voltage regulators were reviewed. For transformers and voltage regulators, different connections were considered.

A unified, three-phase branch model as a generalization of the existing three-phase line models was presented. Using this approach, it is possible to model voltage regulators or tap changers and three-phase conductors or transformers on the same branch without the introduction of an extra support bus. Thus, the system dimension, i.e. the size of the problem, is not increased in the modeling phase. Moreover, the computer program's design is simplified with this model.

Finally, an example was presented that demonstrates the use of a model for several different branch configurations.

Chapter 3

Numerical Solution of the Estimation Problem

The reviewed estimators, MLE, WLS, LAV, and SHGM, are solved by minimizing the objective function they yield (Eqns. 2.7, 2.10, 2.11, and 2.12). Since the objective function is generally non-linear, there is no closed-form solution. In such a case, we need to resort to iterative methods. Typical examples include the Newton-Raphson method and the Gauss-Newton method. In general, these methods will converge on a global minimum if the initial guess is close to the true minimum. If not, convergence may not be attained, or the method may converge to a local minimum. The difficulty with the use of iterative methods is that, in general, we do not know beforehand if they will converge and whether the solution produced is the global minimum. A good initial guess is, thus, of paramount importance. In the following sections, the Newton-Raphson, Gauss-Newton, augmented Hachtel, and Iteratively Re-weighted Least Squares numerical methods will be reviewed.

3.1 Newton-Raphson Iterative Method

The Newton-Raphson iterative numerical method can be used for obtaining the minimum of the objective function that is produced by the WLS (and MLE) estimator. The algorithm can be derived by differentiation of the objective function Eqn. 2.10:

$$\frac{\partial J(\mathbf{x})_{WLS}}{\partial x_j} = -2 \sum_{i=1}^m r_i^{-1} (f_i - h_i(\mathbf{x})) \frac{\partial h_i(\mathbf{x})}{\partial x_j} = 0, \quad (3.1)$$

where $j = 1, 2, \dots, n$. The Jacobian matrix with partial derivatives is defined:

$$\left[\frac{\partial \mathbf{h}(\mathbf{x})}{\partial \mathbf{x}} \right]_{i,j} = \frac{\partial h_i(\mathbf{x})}{\partial x_j}, \quad (3.2)$$

where $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$. The necessary conditions in vector form are then:

$$\frac{\partial J(\mathbf{x})_{WLS}}{\partial \mathbf{x}} = \frac{\partial \mathbf{h}(\mathbf{x})^T}{\partial \mathbf{x}} \mathbf{R}^{-1} (\mathbf{f} - \mathbf{h}(\mathbf{x})) = \mathbf{0}. \quad (3.3)$$

Eqn. 3.3 represents a set of m non-linear equations. To obtain the solution, the iteration scheme is constructed as follows, we define:

$$\mathbf{g}(\mathbf{x}) = \frac{\partial \mathbf{h}(\mathbf{x})^T}{\partial \mathbf{x}} \mathbf{R}^{-1} (\mathbf{f} - \mathbf{h}(\mathbf{x})) \quad (3.4)$$

the (Newton-Raphson) iteration is then:

$$\mathbf{x}_{k+1} = \mathbf{x}_k - \left(\frac{\partial \mathbf{g}(\mathbf{x}_k)}{\partial \mathbf{x}_k} \right)^{-1} \mathbf{g}(\mathbf{x}_k). \quad (3.5)$$

The Jacobian of $\mathbf{g}$ is:

$$\frac{\partial [\mathbf{g}(\mathbf{x})]_i}{\partial x_j} = \frac{\partial}{\partial x_j} \left[\sum_{p=1}^m r_p^{-1} (f_p - h_p(\mathbf{x})) \frac{\partial h_p(\mathbf{x})}{\partial x_i} \right] \quad (3.6)$$

$$= \sum_{p=1}^m \left[r_p^{-1} (f_p - h_p(\mathbf{x})) \frac{\partial^2 h_p(\mathbf{x})}{\partial x_i \partial x_j} - r_p^{-1} \frac{\partial h_p(\mathbf{x})}{\partial x_i} \frac{\partial h_p(\mathbf{x})}{\partial x_j} \right]. \quad (3.7)$$

In vector form, the Jacobian is:

$$\frac{\partial \mathbf{g}(\mathbf{x})}{\partial \mathbf{x}} = \sum_{p=1}^m \mathbf{G}_p(\mathbf{x}) r_p^{-1} (f_p - h_p(\mathbf{x})) - \mathbf{H}^T(\mathbf{x}) \mathbf{R}^{-1} \mathbf{H}(\mathbf{x}) \quad (3.8)$$

where matrices $\mathbf{H}(\mathbf{x})$, $\mathbf{G}_p(\mathbf{x})$ are:

$$[\mathbf{H}(\mathbf{x})]_{i,j} = \left[\frac{\partial \mathbf{g}(\mathbf{x})}{\partial \mathbf{x}} \right]_{i,j} = \frac{\partial h_i(\mathbf{x})}{\partial x_j}, \quad (3.9)$$

$$[\mathbf{G}_p(\mathbf{x})]_{i,j} = \frac{\partial^2 h_i(\mathbf{x})}{\partial x_i \partial x_j}. \quad (3.10)$$

Considering Eqn. 3.5, Eqn. 3.8, and Eqn. 3.4, the final form of the Newton-Raphson iteration is:

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \left(\mathbf{H}^T(\mathbf{x}_k) \mathbf{R}^{-1} \mathbf{H}(\mathbf{x}_k) - \sum_{p=1}^m \mathbf{G}_p(\mathbf{x}_k) r_p^{-1} (f_p - h_p(\mathbf{x}_k)) \right)^{-1} \mathbf{H}^T(\mathbf{x}_k) \mathbf{R}^{-1} (\mathbf{f} - \mathbf{h}(\mathbf{x}_k)). \quad (3.11)$$

The main difficulty of the Newton-Raphson method is the calculation of the matrix of second derivatives $\mathbf{G}_p$ of measurement functions. In general, the method has a quadratic rate of convergence. The convergence, however, is not guaranteed and depends heavily on the selection of the initial guess. In our implementation, for medium sized distribution networks (IEEE distribution feeders up to 123 buses) with moderate loading, the selection of the flat-start (i.e. phase angles of an unloaded feeder and nominal voltage magnitudes) initial solution was usually suitable. Initial solution needs to account for phase shifts due to transformer connections, and magnitude scaling due to tap changers (see section 3.5). Measurement configurations with a prevailing number of power flow measurements can have convergence problems with the flat-start initial solution; because real and reactive power flows are nearly zero at the flat-start, the Jacobian matrix can become ill-conditioned (or even singular) during the first iteration. One of the possible solutions for this condition is explained in [38].

Except for the possible convergence problems that can be solved with a better choice of the initial solution in majority of cases, and the difficulty of computation of the second derivatives of the measurement functions, the Newton-Raphson iteration efficiently delivers a solution to a state estimation problem.

3.2 Gauss-Newton Iteration Method

The Gauss-Newton method is a modification of the Newton-Raphson method. In comparison with the Newton-Raphson method, it does not require second-order derivatives of the measurement functions. The tradeoff is that it has slower rate of convergence. The method can suffer from the same convergence problems, if the initial solution is poorly selected or in the case where measurement configuration contains a prevailing number of power flow measurements.

The method can be derived as follows. The signal model is linearized about some nominal $\mathbf{x}$:

$$\mathbf{h}_p(\mathbf{x}) = \mathbf{h}(\mathbf{x}_0) + \left. \frac{\partial \mathbf{h}_p(\mathbf{x})}{\partial \mathbf{x}} \right|_{\mathbf{x}=\mathbf{x}_0} (\mathbf{x} - \mathbf{x}_0) \quad (3.12)$$

the WLS error becomes:

$$J = \sum_{p=1}^m r_i^{-1} (f_p - h_p(\mathbf{x}))^2 \quad (3.13)$$

$$\approx \sum_{p=1}^n r_i^{-1} \left(f_p - h_p(\mathbf{x}_0) + \left. \frac{\partial h_p(\mathbf{x})}{\partial \mathbf{x}} \right|_{\mathbf{x}=\mathbf{x}_0} \mathbf{x}_0 - \left. \frac{\partial h_p(\mathbf{x})}{\partial \mathbf{x}} \right|_{\mathbf{x}=\mathbf{x}_0} \mathbf{x} \right)^2 \quad (3.14)$$

$$= (\mathbf{f} - \mathbf{h}(\mathbf{x}_0) + \mathbf{H}(\mathbf{x}_0)\mathbf{x}_0 - \mathbf{H}(\mathbf{x}_0)\mathbf{x})^T \mathbf{R}^{-1} (\mathbf{f} - \mathbf{h}(\mathbf{x}_0) + \mathbf{H}(\mathbf{x}_0)\mathbf{x}_0 - \mathbf{H}(\mathbf{x}_0)\mathbf{x}). \quad (3.15)$$

The obtained linearized criterion is solved for $\mathbf{x}$ and the Gauss-Newton iteration scheme is obtained:

$$\mathbf{x}_{k+1} = \mathbf{x}_k + (\mathbf{H}^T(\mathbf{x}_k)\mathbf{R}^{-1}\mathbf{H}(\mathbf{x}_k))^{-1}\mathbf{H}^T(\mathbf{x}_k)\mathbf{R}^{-1}(\mathbf{f} - \mathbf{h}(\mathbf{x}_k)). \quad (3.16)$$

The Gauss-Newton iteration scheme is depicted in Fig. 3.1. It is evident that the Gauss-Newton iteration scheme is based on successive linearizations followed by solving a linear least squares problems. Linear subproblems are solved by the use of the product $\mathbf{H}^T(\mathbf{x}_k)\mathbf{H}(\mathbf{x}_k)$ which maintains the sparsity of the original matrix, but is intrinsically ill-conditioned [47]. To this end, methods were developed that circumvent this difficulty, such as augmented matrix method reviewed in 3.3.

3.3 Hachtel's Augmented Matrix Method

The presence of the product of matrix $\mathbf{H}(\mathbf{x}_k)$ with its transposed version $\mathbf{H}^T(\mathbf{x}_k)$ in Eqn. 3.16 has a significant effect on the conditioning of the system. The condition number is a measure of how much the output value of the system (or equation) can change for a small change in the input value. For a matrix, it is calculated as:

$$\kappa(\mathbf{A}) = \|\mathbf{A}\| \times \|\mathbf{A}^{-1}\| \quad (3.17)$$

and it can be shown that:

$$\kappa(\mathbf{A}^T \mathbf{A}) = [\kappa(\mathbf{A})^2]. \quad (3.18)$$

For condition numbers above 10^3 , the system can be treated as ill-conditioned, meaning that to successfully compute the result through the normal equations the condition number should be smaller than that. Specific sources of ill-conditioning were identified in the literature [38]:

- Large weighting factors used along with virtual measurements
- Short and long lines simultaneously present at the same bus

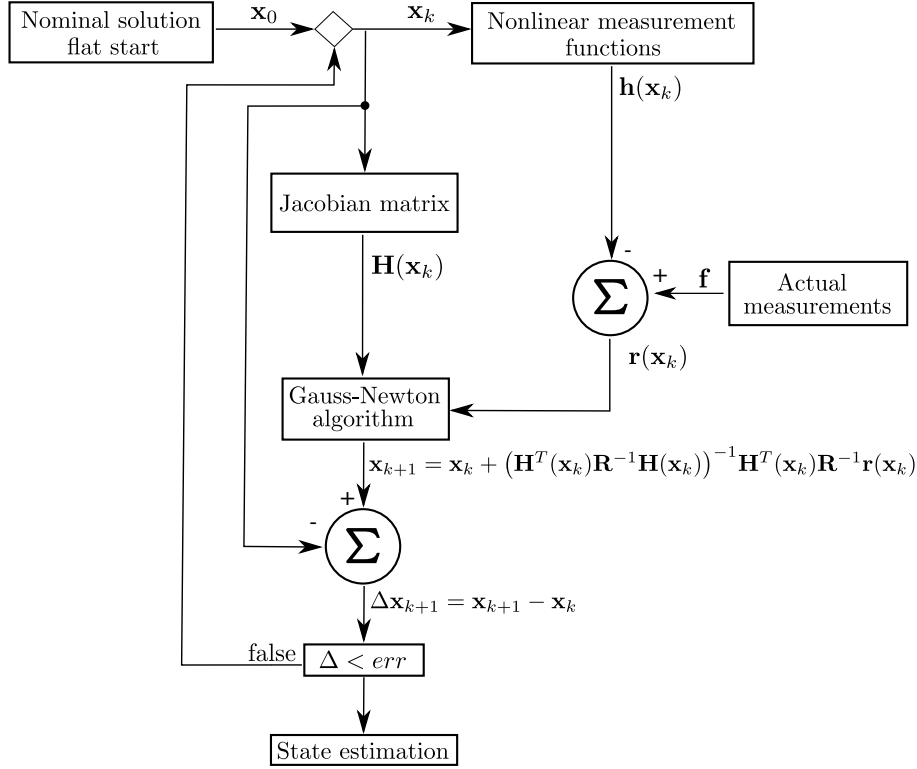


Figure 3.1: Gauss-Newton iteration scheme.

- A large proportion of injection measurements

Conditioning in the presence of the listed sources is further exacerbated as the system grows in size. The problem of solving the least squares problem must, thus, be restated.

Several methods were developed to avoid or neutralize the listed sources of ill-conditioning. Methods using orthogonal transformations, such as those of Householder and Givens, have better stability properties [48], [49]. On the other hand, orthogonal transformations may give a large "fill-in" in sparse matrices [50]. A method that avoids this has been described in [51] but the operation count may be higher than with the normal equations. Based on the comparison of approaches with the normal equations, orthogonal transformations, Hachtel's method, and method of Peters and Wilkinson, reference [50] recommends the use of Hachtel's method for sparse WLS problems.

As the second and third identified sources are broader system design factors that cannot be influenced by the selection of the estimator solution method, the first source, namely the use of large weighting factors used along with virtual measurements, can be eliminated with the implementation of Hachtel's method. Hachtel's method restates the WLS problem as an optimization problem where virtual measurements are presented as explicit optimization constraints. The problem is thus written as:

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & J_{WLS}(\mathbf{x}) = \frac{1}{2}(\mathbf{f} - \mathbf{h}(\mathbf{x}))^T \mathbf{R}^{-1}(\mathbf{f} - \mathbf{h}(\mathbf{x})) \\ \text{subject to} \quad & \mathbf{c}(\mathbf{x}) = \mathbf{0} : \boldsymbol{\lambda}, \end{aligned} \quad (3.19)$$

Where $J_{WLS}(\mathbf{x})$ was multiplied by 1/2 because it simplifies the derivation. The multiplication does not change the objective function in any relevant way. The J_{WLS} is minimized

iteratively as a sequence of linear problems, where in the k -th iteration one minimizes:

$$J_{WLS}(\mathbf{x}) = \frac{1}{2}(\Delta \mathbf{f} - \mathbf{H}\Delta \mathbf{x})^T \mathbf{R}^{-1}(\Delta \mathbf{f} - \mathbf{H}\Delta \mathbf{x}), \quad (3.20)$$

where:

$$\Delta \mathbf{x} = \mathbf{x}_k - \mathbf{x}_{k-1}, \quad (3.21)$$

$$\Delta \mathbf{z} = \mathbf{z} - \mathbf{h}(\mathbf{x}_k), \quad (3.22)$$

$\mathbf{x}_k$ = solution in the k -th iteration,

$\mathbf{H}$ = Jacobian matrix evaluated at $\mathbf{x}_k$.

In order to include the nonlinear equality constraints from Eqn. 3.19 in a solution algorithm, they need to be linearized:

$$\mathbf{C}\Delta \mathbf{x} = \Delta \mathbf{c} \quad (3.23)$$

where $\mathbf{C}$ is a o by n matrix of first derivatives:

$$[\mathbf{C}]_{i,j} = \left[\frac{\partial \mathbf{c}(\mathbf{x})}{\partial \mathbf{x}} \right]_{i,j} = \frac{\partial c_i(\mathbf{x})}{\partial x_j}, \quad (3.24)$$

and $\Delta \mathbf{c} = -\mathbf{c}(\mathbf{x}_k)$ compensates for the linearization error in the previous iteration. The Lagrangian is then constructed as:

$$\mathcal{L} = \frac{1}{2}(\Delta \mathbf{f} - \mathbf{H}\Delta \mathbf{x})^T \mathbf{R}^{-1}(\Delta \mathbf{f} - \mathbf{H}\Delta \mathbf{x}) - \boldsymbol{\lambda}^T(\Delta \mathbf{c} - \mathbf{C}\Delta \mathbf{x}). \quad (3.25)$$

The minimum is found by deriving with respect to $\mathbf{x}$:

$$\frac{\partial \mathcal{L}}{\partial \mathbf{x}} = \mathbf{H}^T(\Delta \mathbf{z} - \mathbf{H}\Delta \mathbf{x}) - \mathbf{C}^T \boldsymbol{\lambda} = \mathbf{0}. \quad (3.26)$$

The residual vector $\mathbf{r}$ is treated as an unknown:

$$\mathbf{r} = \Delta \mathbf{z} - \mathbf{H}\Delta \mathbf{x} \quad (3.27)$$

Eqn. 3.23, Eqn. 3.26, and Eqn. 3.27 are assembled into a matrix equation:

$$\begin{bmatrix} \alpha \mathbf{R} & \mathbf{H} & \mathbf{0} \\ \mathbf{H}^T & \mathbf{0} & \mathbf{C}^T \\ \mathbf{0} & \mathbf{C} & \mathbf{0}^T \end{bmatrix} \cdot \begin{bmatrix} \alpha^{-1} \mathbf{r} \\ \Delta \mathbf{x} \\ -\alpha^{-1} \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \Delta \mathbf{z} \\ \mathbf{0} \\ \Delta \mathbf{c} \end{bmatrix} \quad (3.28)$$

Eqn. 3.28 is an augmented matrix formulation of the linear WLS problem with equality constraints. The system has $m+n+o$ unknowns, and the problem dimension and thus computational load is thus increased, compared to the Newton-Raphson and Gauss-Newton methods with n unknowns. However, the system remains sparse, and this property can be beneficially leveraged in the implementation. Conditioning of the system is improved significantly, as the computation of product $\mathbf{H}^T \mathbf{H}$ is avoided.

Factor α in Eqn. 3.28 cancels out mathematically, but it influences the conditioning of the equation. In our case, we used the following value [38]:

$$\alpha = \max R_{ii}^{-1}. \quad (3.29)$$

Comparison studies [38] show that, for a WLS estimator, Hachtel's method is the cheapest and best conditioned approach for realistic power networks.

3.4 Iteratively Re-Weighted Least Squares

The solutions to SHGM and LAV estimators can be obtained by different methods. The LAV estimator is actually a linear programming problem and can be solved by a plethora of methods discussed in the literature [52]. The Newton-Raphson method requires a computation of second derivatives of the $|f - h_i(\mathbf{x})|$ and $\gamma_i^2 \rho(r_i)$ functions for the LAV and SHGM estimators, respectively. A method that does not require second-order derivatives and is suitable for the solution of both estimators is the Iteratively Re-weighted Least Squares (IRLS) method.

IRLS solves the following problem:

$$\underset{\mathbf{x}}{\operatorname{argmin}} \sum_{i=1}^n \left| f_i - h_i(\mathbf{x}) \right|^p, \quad (3.30)$$

by an iterative method in which each step involves solving a weighted least squares problem of the form:

$$\mathbf{x}^{k+1} = \underset{\mathbf{x}}{\operatorname{argmin}} \sum_{i=1}^n w_i \left(f_i - h_i(\mathbf{x}) \right)^2. \quad (3.31)$$

A correction factor as a weight w_i is introduced, and $(f_i - h_i(\mathbf{x}))^2$ is minimized using WLS. The initial weight factors are chosen as $w_i = 1$, and in subsequent iterations they are calculated as:

$$w_i^{k+1} = w_i^k \left| f_i - h_i(\mathbf{x}) \right|^{p-2}. \quad (3.32)$$

It was shown that the IRLS method is less prone to numerical problems than the Newton-Raphson method in the case of the SHGM estimator [39]. It was also shown that IRLS almost always converges on a solution in a few iterations from the flat-start initial solution [39].

3.5 Relation to Our Network Model

The discussed numerical methods were implemented in software using a combination of C++ and MATLAB tools. Besides the solution method itself, the implementation requires an algorithm for a computation of the initial solution, first order partial derivatives for all the measurement functions, and a configuration of measurements that makes the power distribution system observable.

3.5.1 Determination of the initial solution

In our implementation, the flat-start solution is chosen as the initial solution. The solution assumes nominal voltage magnitudes (i.e. 1 per unit) and phase angles of an unloaded feeder. In determination of the initial phase angles, phase shifts that occur due to different transformer connections need to be considered. The Algorithm 3.1 presents a recursive algorithm that calculates the initial phase angles for a radial feeder.

Hachtel's method, as well as the Gauss-Newton and Newton-Raphson methods, require first-order partial derivatives with respect to the measurement functions. The derivatives of the measurement functions were computed analytically, and are presented in the following subsections.

Algorithm 3.1: Algorithm for the computation of initial phase angles

Data: Vector of phase angles, Bus ID**Result:** Vector of initial phase angles

current value = angle + node phase rotation(Bus ID);

initial value(Bus ID) = current value;

foreach *child in Bus ID* **do** call this method (vector of phase angles, child node ID);**3.5.2 Partial derivatives for power flow equations**

$$\begin{aligned} \frac{\partial P_{km}^a}{\partial |V_k^a|} = & \sum_{j \in \Omega_p} \sum_{n \in \Omega_p} \sum_{i \in \Omega_p} \left\{ \left[(g_{km}^{jn} + g_{km,sh}^{jn}) t_{km}^{ni} t_{km}^{aj} |V_k^i| \cos(\phi_k^a - \phi_k^i) \right. \right. \\ & + (b_{km}^{jn} + b_{km,sh}^{jn}) t_{km}^{ni} t_{km}^{aj} |V_k^i| \sin(\phi_k^a - \phi_k^i) \Big] - \left[g_{km}^{jn} t_{km}^{aj} t_{km}^{ni} |V_m^i| \cos(\phi_k^a - \phi_m^i) \right. \\ & \left. \left. - g_{km}^{jn} t_{km}^{aj} t_{km}^{ni} |V_m^i| \sin(\phi_k^a - \phi_m^i) \right] \right\} + \sum_{j \in \Omega_p} \sum_{n \in \Omega_p} |V_k^a| t_{km}^{na} t_{km}^{aj} (g_{km}^{jn} + g_{km,sh}^{jn}) \quad (3.33) \end{aligned}$$

$$\frac{\partial P_{km}^a}{\partial |V_m^i|} = -|V_k^a| \sum_{j \in \Omega_p} \sum_{n \in \Omega_p} \left[g_{km}^{jn} t_{km}^{aj} t_{km}^{ni} \cos(\phi_k^a - \phi_m^i) + b_{km}^{jn} t_{km}^{ni} t_{km}^{aj} \sin(\phi_k^a - \phi_m^i) \right] \quad (3.34)$$

$$\begin{aligned} \frac{\partial P_{km}^a}{\partial |V_k^o|} = & |V_k^a| \sum_{j \in \Omega_p} \sum_{n \in \Omega_p} \left[(g_{km}^{jn} + g_{km,sh}^{jn}) t_{km}^{ni} t_{km}^{aj} \cos(\phi_k^a - \phi_k^i) + (b_{km}^{jn} + b_{km,sh}^{jn}) t_{km}^{ni} \right. \\ & \left. t_{km}^{aj} \sin(\phi_k^a - \phi_k^i) \right] \quad (3.35) \end{aligned}$$

$$\begin{aligned} \frac{\partial P_{km}^a}{\partial \phi_k^a} = & |V_k^a| \sum_{j \in \Omega_p} \sum_{n \in \Omega_p} \sum_{i \in \Omega_p} \left\{ |V_k^i| \left[(g_{km}^{jn} + g_{km,sh}^{jn}) t_{km}^{ni} t_{km}^{aj} \sin(\phi_k^a - \phi_k^i) \right. \right. \\ & + (b_{km}^{jn} + b_{km,sh}^{jn}) t_{km}^{ni} t_{km}^{aj} \cos(\phi_k^a - \phi_k^i) \Big] - |V_m^i| \left[-g_{km}^{jn} t_{km}^{ni} t_{km}^{aj} \sin(\phi_k^a - \phi_m^i) \right. \\ & \left. \left. + b_{km}^{jn} t_{km}^{ni} t_{km}^{aj} \cos(\phi_k^a - \phi_m^i) \right] \right\} - |V_k^a| \sum_{j \in \Omega_p} \sum_{n \in \Omega_p} (b_{km}^{jn} + b_{km,sh}^{jn}) t_{km}^{na} t_{km}^{aj} |V_k^a| \quad (3.36) \end{aligned}$$

$$\begin{aligned} \frac{\partial P_{km}^a}{\partial \phi_m^i} = & -|V_k^a| \sum_{j \in \Omega_p} \sum_{n \in \Omega_p} \sum_{i \in \Omega_p} |V_m^i| \left[-g_{km}^{jn} t_{km}^{ni} t_{km}^{aj} \sin(\phi_k^a - \phi_m^i) - b_{km}^{jn} t_{km}^{ni} t_{km}^{aj} \right. \\ & \left. \cos(\phi_k^a - \phi_m^i) \right] \quad (3.37) \end{aligned}$$

$$\begin{aligned} \frac{\partial P_{km}^a}{\partial \phi_k^i} = & |V_k^a| \sum_{j \in \Omega_p} \sum_{n \in \Omega_p} |V_k^i| \left[(g_{km}^{jn} + g_{km,sh}^{jn}) t_{km}^{ni} t_{km}^{aj} \sin(\phi_k^a - \phi_k^i) - (b_{km}^{jn} + b_{km,sh}^{jn}) \right. \\ & \left. t_{km}^{ni} t_{km}^{aj} \cos(\phi_k^a - \phi_k^i) \right] \quad (3.38) \end{aligned}$$

$$\begin{aligned}
\frac{\partial Q_{km}^a}{\partial |V_k^a|} = & \sum_{j \in \Omega_p} \sum_{n \in \Omega_p} \sum_{i \in \Omega_p} \left\{ |V_k^i| \left[(g_{km}^{jn} + g_{km,sh}^{jn}) t_{km}^{ni} t_{km}^{aj} \sin(\phi_k^a - \phi_k^i) \right. \right. \\
& - (b_{km}^{jn} + b_{km,sh}^{jn}) t_{km}^{ni} t_{km}^{aj} \cos(\phi_k^a - \phi_k^i) \left. \right] - |V_m^i| \left[g_{km}^{jn} t_{km}^{aj} t_{mk}^{ni} \sin(\phi_k^a - \phi_m^i) \right. \\
& \left. \left. - g_{km}^{jn} t_{km}^{aj} t_{mk}^{ni} \cos(\phi_k^a - \phi_m^i) \right] \right\} - \sum_{j \in \Omega_p} \sum_{n \in \Omega_p} |V_k^a| t_{km}^{na} t_{km}^{aj} (b_{km}^{jn} + b_{km,sh}^{jn}) \quad (3.39)
\end{aligned}$$

$$\frac{\partial Q_{km}^a}{\partial |V_m^i|} = -|V_k^a| \sum_{j \in \Omega_p} \sum_{n \in \Omega_p} \left[g_{km}^{jn} t_{km}^{aj} t_{mk}^{ni} \sin(\phi_k^a - \phi_m^i) - b_{km}^{jn} t_{mk}^{ni} t_{km}^{aj} \cos(\phi_k^a - \phi_m^i) \right] \quad (3.40)$$

$$\begin{aligned}
\frac{\partial Q_{km}^a}{\partial |V_k^o|} = & |V_k^a| \sum_{j \in \Omega_p} \sum_{n \in \Omega_p} \left[(g_{km}^{jn} + g_{km,sh}^{jn}) t_{km}^{ni} t_{km}^{aj} \sin(\phi_k^a - \phi_k^i) - (b_{km}^{jn} + b_{km,sh}^{jn}) \right. \\
& \left. t_{km}^{ni} t_{km}^{aj} \cos(\phi_k^a - \phi_k^i) \right] \quad (3.41)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial Q_{km}^a}{\partial \phi_k^a} = & |V_k^a| \sum_{j \in \Omega_p} \sum_{n \in \Omega_p} \sum_{i \in \Omega_p} \left\{ |V_k^i| \left[(g_{km}^{jn} + g_{km,sh}^{jn}) t_{km}^{ni} t_{km}^{aj} \cos(\phi_k^a - \phi_k^i) \right. \right. \\
& + (b_{km}^{jn} + b_{km,sh}^{jn}) t_{km}^{ni} t_{km}^{aj} \sin(\phi_k^a - \phi_k^i) \left. \right] - |V_m^i| \left[-g_{km}^{jn} t_{mk}^{ni} t_{km}^{aj} \cos(\phi_k^a - \phi_m^i) \right. \\
& \left. \left. + b_{km}^{jn} t_{mk}^{ni} t_{km}^{aj} \sin(\phi_k^a - \phi_m^i) \right] \right\} - |V_k^a|^2 \sum_{j \in \Omega_p} \sum_{n \in \Omega_p} (g_{km}^{jn} + g_{km,sh}^{jn}) t_{km}^{na} t_{km}^{aj} \quad (3.42)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial Q_{km}^a}{\partial \phi_m^i} = & -|V_k^a| \sum_{j \in \Omega_p} \sum_{n \in \Omega_p} \sum_{i \in \Omega_p} |V_m^i| \left[-g_{km}^{jn} t_{mk}^{ni} t_{km}^{aj} \cos(\phi_k^a - \phi_m^i) + b_{km}^{jn} t_{mk}^{ni} t_{km}^{aj} \right. \\
& \left. \sin(\phi_k^a - \phi_m^i) \right] \quad (3.43)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial Q_{km}^a}{\partial \phi_k^i} = & |V_k^a| \sum_{j \in \Omega_p} \sum_{n \in \Omega_p} |V_k^i| \left[(g_{km}^{jn} + g_{km,sh}^{jn}) t_{km}^{ni} t_{km}^{aj} \cos(\phi_k^a - \phi_k^i) - (b_{km}^{jn} + b_{km,sh}^{jn}) \right. \\
& \left. t_{km}^{ni} t_{km}^{aj} \sin(\phi_k^a - \phi_k^i) \right] \quad (3.44)
\end{aligned}$$

where $i \in \{a, b, c\}$ and $o \in \{b, c\}$.

3.5.3 Partial derivatives for power injection equations

$$\frac{\partial P_k^a}{\partial |V_k^a|} = \sum_{m \in \kappa} \sum_{p \in \Omega_p} |V_m^p| (G_{km}^{ap} \cos(\phi_k^a - \phi_m^p) + B_{km}^{ap} \sin(\phi_k^a - \phi_m^p)) + |V_k^a| G_{kk}^{aa} \quad (3.45)$$

$$\frac{\partial P_k^a}{\partial |V_m^p|} = |V_k^a| (G_{km}^{ap} \cos(\phi_k^a - \phi_m^p) + B_{km}^{ap} \sin(\phi_k^a - \phi_m^p)) \quad (3.46)$$

$$\begin{aligned} \frac{\partial P_k^a}{\partial \phi_k^a} &= |V_k^a| \sum_{m \in \kappa} \sum_{p \in \Omega_p} |V_m^p| (-G_{km}^{ap} \sin(\phi_k^a - \phi_m^p) + B_{km}^{ap} \cos(\phi_k^a - \phi_m^p)) \\ &\quad - |V_k^a|^2 B_{kk}^{aa} \end{aligned} \quad (3.47)$$

$$\frac{\partial P_k^a}{\partial \phi_m^p} = |V_k^a| |V_m^p| (G_{km}^{ap} \sin(\phi_k^a - \phi_m^p) - B_{km}^{ap} \cos(\phi_k^a - \phi_m^p)) \quad (3.48)$$

$$\frac{\partial Q_k^a}{\partial |V_k^a|} = \sum_{m \in \kappa} \sum_{p \in \Omega_p} |V_m^p| (G_{km}^{ap} \sin(\phi_k^a - \phi_m^p) - B_{km}^{ap} \cos(\phi_k^a - \phi_m^p)) - |V_k^a| B_{kk}^{aa} \quad (3.49)$$

$$\frac{\partial Q_k^a}{\partial |V_m^p|} = |V_k^a| (G_{km}^{ap} \sin(\phi_k^a - \phi_m^p) - B_{km}^{ap} \cos(\phi_k^a - \phi_m^p)) \quad (3.50)$$

$$\frac{\partial Q_k^a}{\partial \phi_k^a} = |V_k^a| \sum_{m \in \kappa} \sum_{p \in \Omega_p} |V_m^p| (G_{km}^{ap} \cos(\phi_k^a - \phi_m^p) + B_{km}^{ap} \sin(\phi_k^a - \phi_m^p)) - |V_k^a|^2 G_{kk}^{aa} \quad (3.51)$$

$$\frac{\partial Q_k^a}{\partial \phi_m^p} = |V_k^a| |V_m^p| (-G_{km}^{ap} \cos(\phi_k^a - \phi_m^p) - B_{km}^{ap} \sin(\phi_k^a - \phi_m^p)) \quad (3.52)$$

Where $p \in \{a, b, c\}$ if $m \neq k$, and $p \in \{b, c\}$ if $m = k$.

3.5.4 Generation of measurement configurations

To make the power system observable, a measurement configuration needs to be constructed. A measurement configuration includes the topological locations and measurement types for the power system in question. Several different measurement configurations were tested in simulated scenarios. Measurement configurations include combinations of one or more of the following measurement types:

- Active or reactive branch power flow measurements,
- Active or reactive bus power injection measurements,
- Bus voltage phase measurements,
- Bus voltage magnitude measurements.

In addition to different measurement types, a particular measurement configuration can include active and reactive bus power zero injection constraints. Zero injection constraints are introduced on the buses where we are certain there is no power injection. If this information was modeled as a measurement with a correspondingly high weight (as the information is certain), it would negatively influence the conditioning of the system.

Measurements were generated from the load-flow results for the reference feeder loading. Measurement that has added noise was calculated as follows:

$$z_{\text{meas}} = z_{\text{ref}} + \mathcal{N}(0, z_{\text{ref}} \cdot \frac{z_{\text{acc}}}{2}). \quad (3.53)$$

Measurement device accuracies were modeled as 2 sigma values for the Gaussian random generator, and the values for different devices were selected as follows:

- 10% of the reference value for active and reactive power flow measurements.
- 20% of the reference value for active and reactive power injection measurements.
- 1% of the reference value for the PMU magnitude.
- 0.005° for the PMU angle.

An example of input data (PMU measurements) is depicted in Fig. 3.2. For each measurement configuration, 100 Monte-Carlo runs were performed.

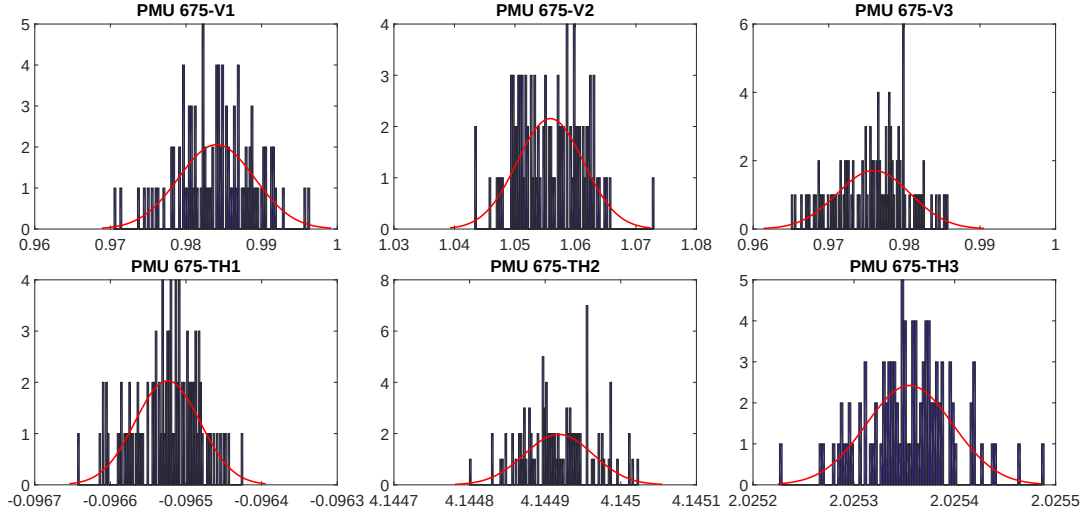


Figure 3.2: Example of input PMU measurements.

3.6 Performance Evaluation on the Reference IEEE Power Distribution feeders

The implementation of numerical solvers was tested on the standard IEEE distribution test feeders [53] with 13 and 123 buses. Measurements were generated from the load-flow results for the reference feeder loading provided by the IEEE [53] (available in Appendix B). On the 13-bus feeder (depicted in Fig. 3.3), the following measurement configurations were tested:

1. Power injection measurements on every bus. No other measurements.
2. Voltage phasor measurements on every bus. No other measurements.
3. Power-flow measurements on every branch. No other measurements.
4. Measurement configuration is listed in Table 3.1.

For each of the listed measurement configurations, 100 Monte-Carlo (MC) simulations were generated. MC simulations were run on all the reviewed estimators (LAV, WLS, and SHGM), where the expected values of the state estimation results were compared to the load-flow results that were used for the generation of measurements.

Figs. 3.4-3.5 depict the voltage phase and magnitude errors (bias and standard deviation) of the SE results for the first measurement configuration and all three estimators. Load-flow results are considered as true values. It is seen that results among different

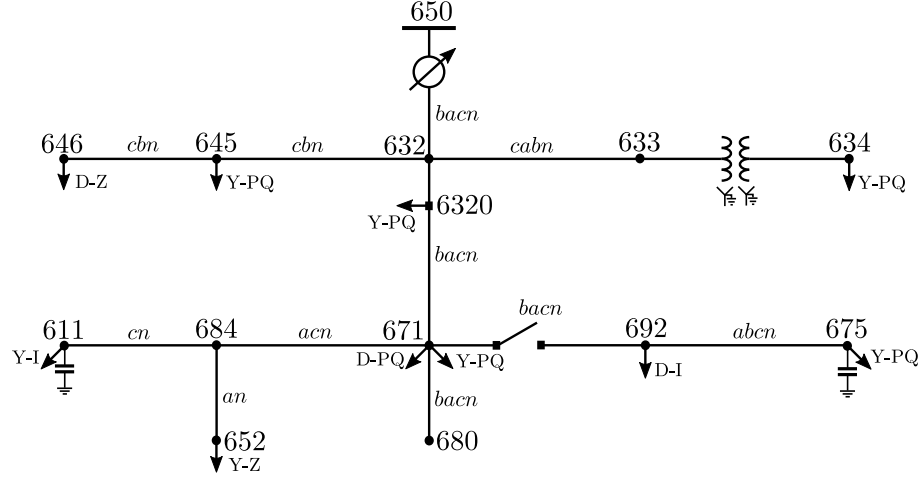


Figure 3.3: IEEE 13-bus feeder.

Table 3.1: Mixed measurement configuration for validation.

Bus / Branch	PQ	PMU	Flow	Zero constraint
632–633		•	•	•
633–634		•	•	•
634		•	•	
645–632		•	•	
646–645		•	•	
6320–632		•	•	
671–6320	•	•	•	
692–671	•	•	•	
675–692		•	•	
680–671		•	•	•
684–671		•	•	•
652–684		•	•	
611–684		•	•	

estimators are practically identical. It is also seen that all three estimators yield a true value on average, which aligns with the fact that estimators are unbiased.

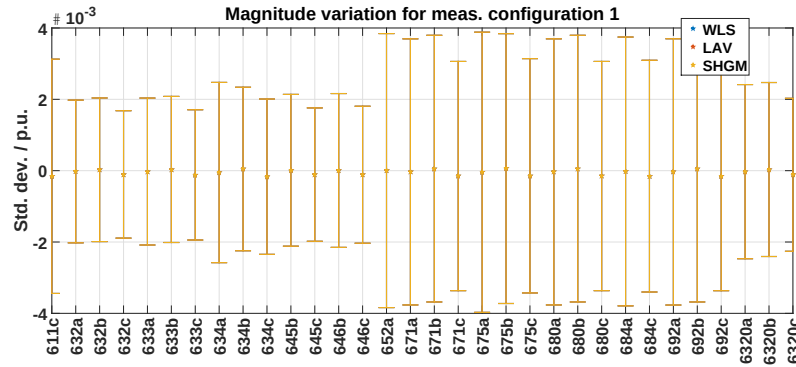


Figure 3.4: Magnitude variation for measurement configuration 1.

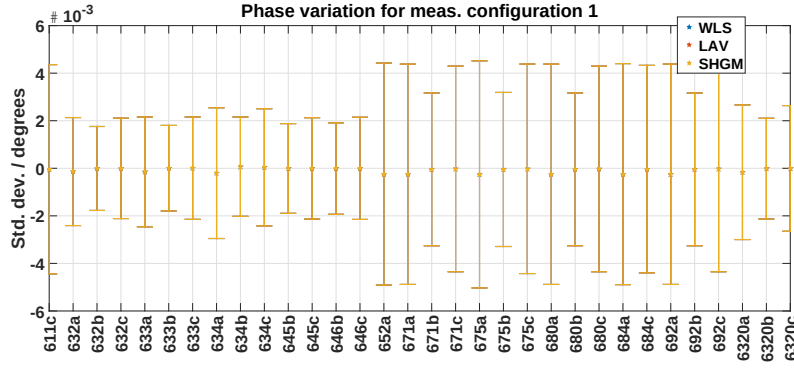


Figure 3.5: Phase variation for measurement configuration 1.

Figs. 3.6-3.9 show a practically identical behavior of state estimators for measurement configurations two and three.

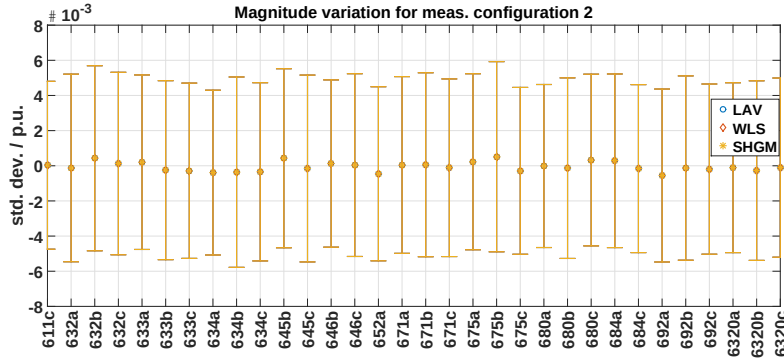


Figure 3.6: Magnitude variation for measurement configuration 2.

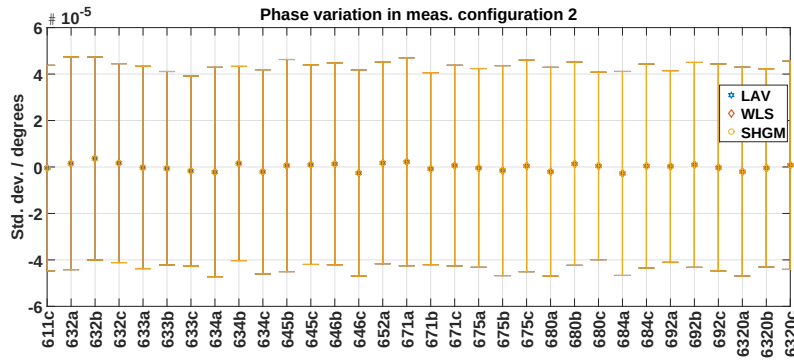


Figure 3.7: Phase variation for measurement configuration 2.

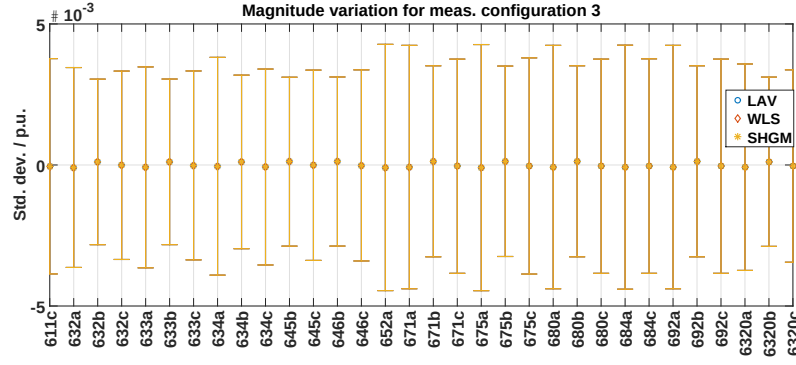


Figure 3.8: Magnitude variation for measurement configuration 3.

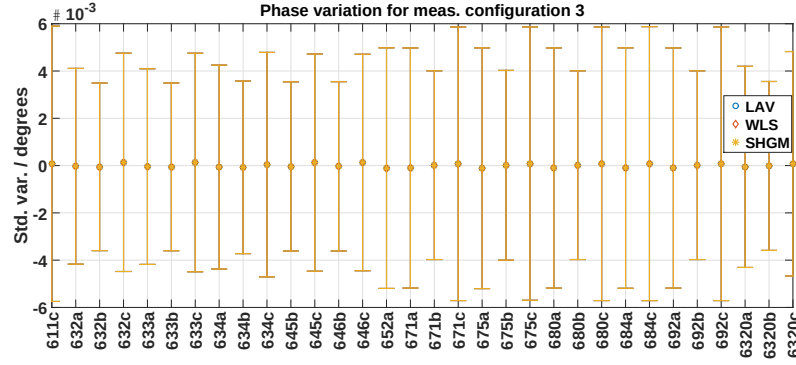


Figure 3.9: Phase variation for measurement configuration 3.

The fourth (mixed) measurement configuration yields more interesting results. From Figs. 3.10 and 3.11, it is seen that the WLS estimator severely outperforms LAV and SHGM in terms of result variance.

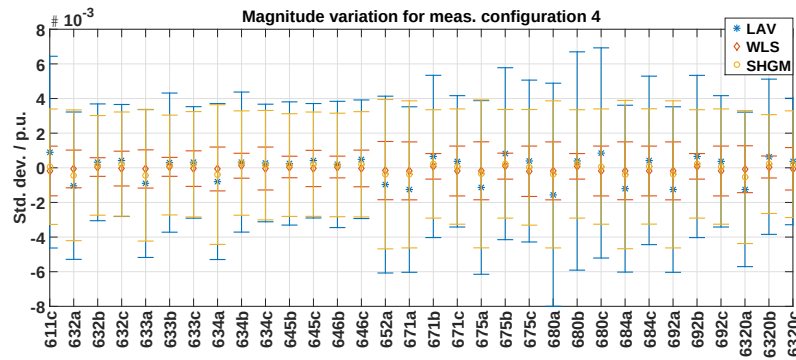


Figure 3.10: Magnitude variation for measurement configuration 4.

Table 3.2: Measurement configurations for IEEE 13 bus feeder.

Meas. config.	2		3	4	5	6	7	8	9
Buses	PQ	Zero const.	PMU	PMU	PMU	PMU	PMU	PMU	PMU
632		•							•
633		•							•
634								•	
645								•	
646							•		
6320							•		
671	•					•			
692	•					•			
675					•				
680		•		•					
684		•		•					
652			•						
611			•						

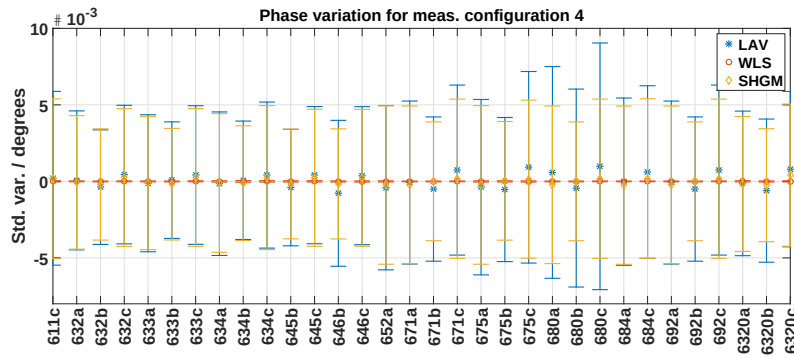


Figure 3.11: Phase variation for measurement configuration 4.

To observe the behavior result variance when measurements are added to the configuration, the following experiment was conducted. The third measurement configuration (flow measurements only) was selected as a starting point, then another eight measurement configurations were created, where in each, measurements and equality constraints were added to the previous one. Table 3.2 lists the configurations and the added measurements. For each configuration, 100 Monte Carlo runs were calculated, with measurement accuracies described in the previous subsection. For every measurement configuration, an average standard deviation for phase and magnitude was calculated. The results of average standard deviations for the listed measurement configurations are depicted in Figs. 3.13 and 3.12. It is seen that added measurements cause the result variance to decrease in the case of the WLS estimator. However, this is not the case with LAV and SHGM estimators, where it is seen that the performance of LAV and SHGM estimators is not significantly influenced by the addition of measurements. The result implies that the WLS estimator is statistically more efficient than LAV and SHGM.

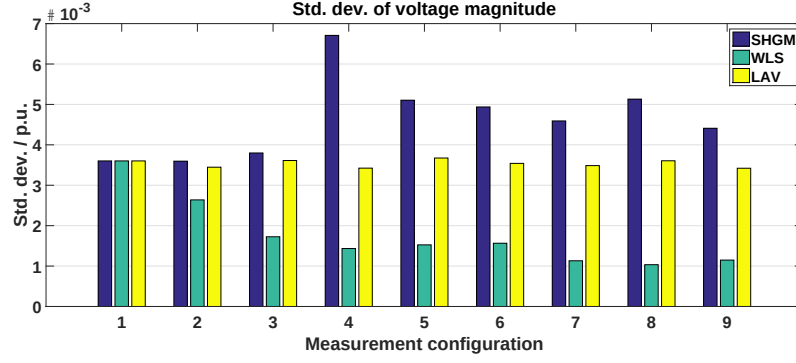


Figure 3.12: Magnitude variation for measurement conf. listed in Table 3.2.

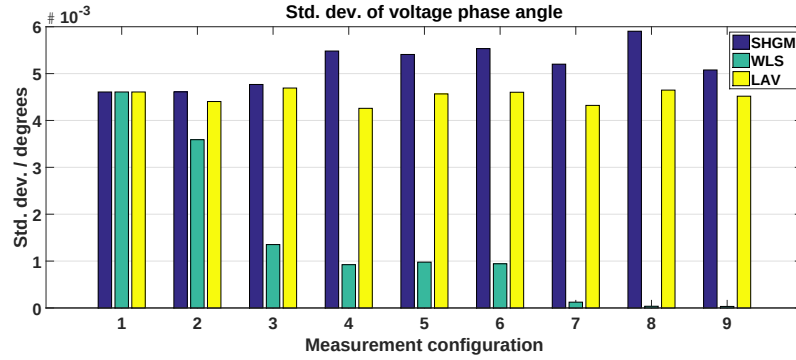


Figure 3.13: Phase variation for measurement conf. listed in Table 3.2.

3.6.1 Computation times of different estimators

In the context of bigger networks, the computational complexity of the estimator is another important aspect. In the discussed estimators, two numerical solution algorithms are used, namely the Iteratively Reweighted Least Squares (IRLS) and Hachtel's method. Besides the algorithm and the network size, the measurement configuration (number, type, and location of measurements) is also an important factor.

Several comparisons were conducted on the IEEE 123 bus feeder (depicted in Fig 3.14). We measured computation times for different estimators (WLS, SHGM, and LAV), and four different measurement configurations. The measurement configurations were as follows:

1. Power injection measurements on every bus. No other measurements.
2. Voltage phasor measurements on every bus. No other measurements.
3. Power-flow measurements on every branch. No other measurements.
4. Measurement configuration is listed in Table A.12.

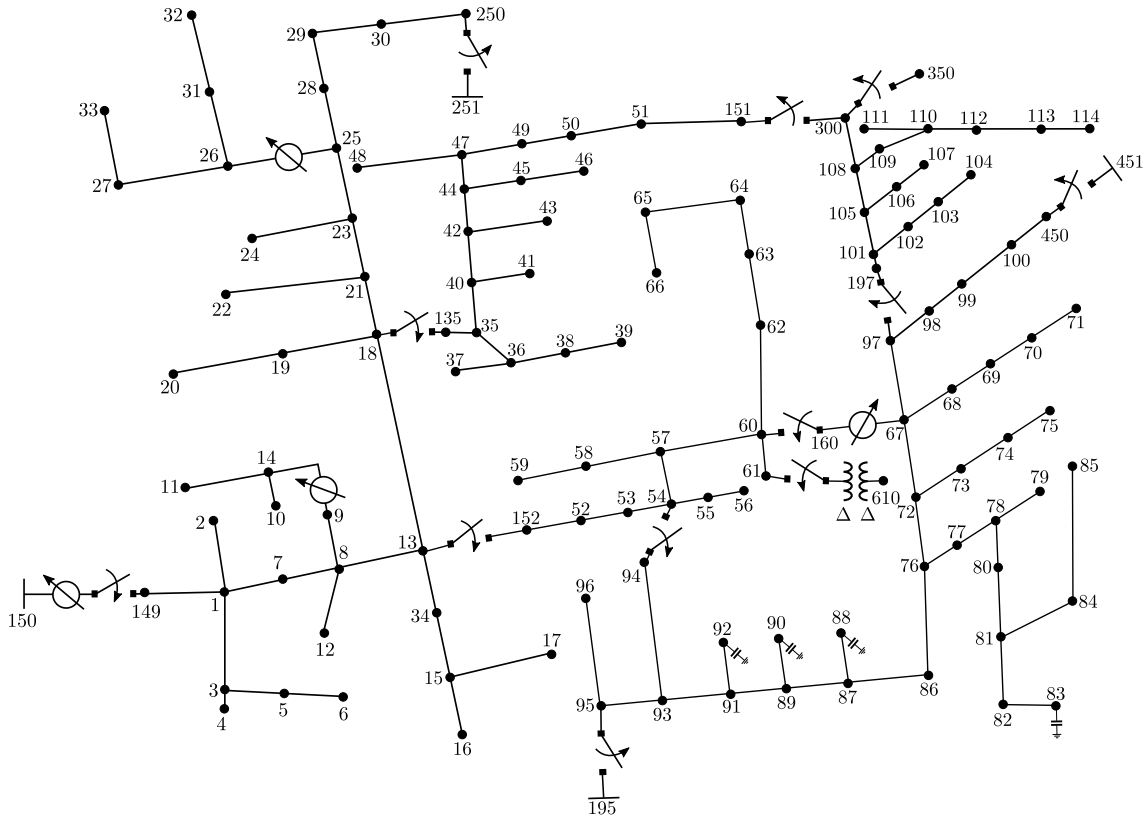


Figure 3.14: IEEE 123-bus feeder.

The measured results are listed in Table 3.3. It is seen that in terms of measurement configurations, the injection measurements pose the greatest burden, followed by the power flow measurements and PMU measurements. Computational times depend on the complexity of measurement expressions which depend on the selection of state variables. In terms of different estimators, it is seen that LAV is generally the fastest, followed by WLS and then SHGM. All three estimators use the Gauss-Newton solver, whereas the LAV and SHGM estimators use the IRLS algorithm on top of it for translation of the original problem into the least squares problem. It must be pointed out that since the estimation

Table 3.3: Computation times

Measurement configuration / Estimator	WLS	LAV	SHGM
PQ injection measurements	3,75	0,57	3,75
Voltage PMU measurements	0,12	0,17	3,00
PQ flow measurements	0,34	0,32	3,20
Configuration 1 in Table A.12	0,58	0,48	3,54

problem is in general non-linear, the computation time does not necessarily scale with the number and type of measurements. To illustrate this fact, an experiment was conducted where the measurement configuration 1 in Table A.12 was taken as a starting configuration, then in each subsequent configuration, selected measurements were added to the previous measurement configuration. Table 3.4 lists the results.

Table 3.4: Computation times

Estimator / Measurement configuration	2	3	4	5	6	7
WLS	0,66	0,69	0,8	0,91	0,88	0,77
LAV	0,52	0,58	0,55	0,66	0,73	0,72
SHGM	3,67	3,69	3,82	3,98	3,83	3,67

3.6.2 Summary

In this chapter, methods for the numerical solution of different state estimators were reviewed. MLE and WLS estimators can be solved directly using the Gauss-Newton or Newton-Raphson method, and SHGM can be solved using the IRLS which also leverages the Gauss-Newton method.

In the Gauss-Newton or Newton-Raphson method, we successively solve the least squares problem with the normal equations. Since the normal equations have a significant effect on the conditioning of the system, Hachtel's augmented matrix method was reviewed that mitigates the intrinsic ill-conditioning of the normal equations.

The contribution of this chapter is the implementation and performance evaluation of different state estimation methods on the proposed unified three-phase branch model. In order to apply the numerical solution to our problem, first, the partial derivatives with respect to the state variables were calculated. Numerical simulations were performed for all estimators, with different measurement configurations on the IEEE test feeder networks with 13 and 123 buses. It was shown that among the three, WLS is the one that most efficiently makes use of the new information, and LAV the one that is least computationally demanding.

Chapter 4

Small Model and Measurement Error Sensitivities

Network model parameters, such as series impedances or shunt admittances (self and mutual) that are derived from topology data (conductor geometry) can be incorrect. This is due to inaccurate data provided by the manufacturer or installation crew, inaccurate measurement campaigns, effects of aging, etc. These issues are even more prevalent in distribution networks, because the networks are larger and harder to keep track off. Uncertain data model has influence over the estimated state variables. Thus, a method that can quickly evaluate sensitivities, i.e. the impact of uncertain parameters on the state estimation results, can be of great benefit.

Several methods were applied to the problem of sensitivity analysis in power system state estimation. In [54], the authors presented a method that compares the covariance matrices of a true state vector based on the error-free model, and a state vector that is based on a model with a known error. The comparison of the estimation bias and variance gives a good measure for evaluating the modeling errors. The drawback of this method is that the complete calculation has to be repeated for every parameter that needs to be evaluated in terms of sensitivity. This quickly becomes unfeasible as the networks become larger than a few dozen buses. In [55], the authors proposed a similar technique that allows the computation of uncertainty sensitivity coefficients for the complete measurement set. Their work, however, does not address the sensitivity calculation for topology parameters and for quantities that are derived from the state variables such as power flows or power injections. In [56], the authors proposed a two-step method for uncertainty analysis. First a weighted least-squares state estimation solution is computed and then the upper and lower interval bounds are obtained using linear programming. If the inaccuracy in the measurements is modeled by probability distribution function, then the calculated state vector is also modeled by a probability distribution function. Since the statistics of measurement errors are difficult to characterize in practice, the idea here is that the accuracy of a particular measurement is described as an interval, for example $\pm 2\%$, rather than by the standard deviation or variance. The second step of the analysis allows the specification of not just a single optimal estimate of the particular state variable, but also an uncertainty range within which one can be certain that the true value lies. The limitation of the method is that the lower and upper bound need to be computed separately for each state variable and for each measurement. In [57], the authors expanded this approach to also include the network parameters. In [58], a similar approach using interval analysis technique was proposed to find the bounds of the state variables of the power system whose line parameters lie within particular bounds. A linear measurement model is assumed (i.e. voltage phasor measurements only), other measurements are included by using the equiva-

lent measurement technique (i.e. measurements are converted into their equivalent voltage phasor values). In [59], [60], the authors developed analytical expressions for the propagation of uncertainty of phasor measurement units (PMUs) in the state estimation algorithm. In [61], the authors also describe the use of the Monte Carlo method for the evaluation of different uncertainty sources on a three-phase state estimator. In [62], Mínguez and Conejo leveraged the perturbation approach to sensitivity analysis based on the differentiation of the Karush-Kuhn-Tucker (KKT) conditions described in [63]. We generalized their work with a full three-phase analysis and with the expression of uncertainties in terms of intervals. Since the calculated sensitivities are only valid at the optimal solution point, we also suggest an upper error bound on the interval analysis.

4.1 Perturbation Approach to Sensitivity Analysis

Sensitivity analysis consists of determining how and how much a certain parameter change in the optimization problem influences the objective function value. The existing methods for sensitivity analysis share the following limitations [63]:

- There are different methods for different cases for obtaining sensitivities, but no integrated approach that yields all the sensitivities at once.
- They assume the existence of the partial derivatives of the objective function with respect to the parameters which is not always the case.
- They assume that the active constraints remain active, which implies that there is no need to distinguish between equality and inequality constraints.

With the perturbation approach, a general analysis can be performed without assuming the existence of the partial derivatives and without assuming that active inequality constraints remain active. Also, the perturbation approach yields all the sensitivities with respect to the parameters at once.

Considering the cost function, the state-estimation problem can be formulated as an optimization problem including constraints:

$$\begin{aligned} & \underset{\mathbf{x}}{\text{minimize}} && J(\mathbf{x}, \mathbf{a}) \\ & \text{subject to} && \mathbf{c}(\mathbf{x}, \mathbf{a}) = 0 : \boldsymbol{\lambda}, \\ & && \mathbf{g}(\mathbf{x}, \mathbf{a}) \leq 0 : \boldsymbol{\mu}, \end{aligned} \tag{4.1}$$

where $\mathbf{x} \in \mathbb{R}^n$, $\mathbf{a} \in \mathbb{R}^p$, $J(\mathbf{x}, \mathbf{a})$ is a scalar cost function, $\mathbf{c}(\mathbf{x}, \mathbf{a})$ is the vector of equality constraints representing exactly known pseudo-measurements (zero injections), $\mathbf{g}(\mathbf{x}, \mathbf{a})$ is the vector of inequality constraints (e.g., the phase angles must be smaller than $\frac{\pi}{2}$), and $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$ are the Lagrange multiplier vectors for the equality and inequality constraints, respectively. Note that vector $\mathbf{a}$ includes measurements and model data that are of interest in the calculation of sensitivities. In our case, it includes all the measurements and all the system-admittance elements, i.e., $\mathbf{G}$, $\mathbf{B}$, $\mathbf{b}_{shunt}$, and $\mathbf{g}_{shunt}$.

4.1.1 Dual variables

Duality is a crucial concept in the following sensitivity analysis of a nonlinear optimization problem. Consider the nonlinear *primal* problem in Eqn. 4.1. The *dual* problem requires the introduction of the dual function defined as:

$$\Theta(\boldsymbol{\lambda}, \boldsymbol{\mu}) = \inf_{\mathbf{x}} \left[J(\mathbf{x}, \mathbf{a}) + \boldsymbol{\lambda}^T \mathbf{c}(\mathbf{x}, \mathbf{a}) + \boldsymbol{\mu}^T \mathbf{g}(\mathbf{x}, \mathbf{a}) \right] \tag{4.2}$$

The *dual* problem is then defined as follows:

$$\begin{aligned} & \underset{\mathbf{x}}{\text{maximize}} && J(\mathbf{x}, \mathbf{a}), \\ & \text{subject to} && \boldsymbol{\mu} \geq 0 \end{aligned} \quad (4.3)$$

Using the Lagrangian:

$$\mathcal{L}(\mathbf{x}, \boldsymbol{\lambda}, \boldsymbol{\mu}) = J(\mathbf{x}, \mathbf{a}) + \boldsymbol{\lambda}^T \mathbf{c}(\mathbf{x}, \mathbf{a}) + \boldsymbol{\mu}^T \mathbf{g}(\mathbf{x}, \mathbf{a}), \quad (4.4)$$

we can rewrite the *dual problem* as:

$$\underset{\boldsymbol{\lambda}, \boldsymbol{\mu}; \boldsymbol{\mu} \geq 0}{\text{maximize}} \left(\underset{\mathbf{x}}{\text{infimum}} \mathcal{L}(\mathbf{x}, \boldsymbol{\lambda}, \boldsymbol{\mu}) \right) \quad (4.5)$$

It is assumed that J , $\mathbf{c}$, and $\mathbf{g}$ are such that the infimum of the Lagrangian function is always attained at some $\mathbf{x}$, so that the infimum operation in Eqn. 4.2 and Eqn. 4.5 can be replaced by the minimum operation. Then, the problem in Eqn. 4.5 is referred to as the *max-min* dual problem [64].

4.1.2 Sensitivities

The sensitivities around the current operating point are of interest, thus, the first step towards obtaining the sensitivities is to calculate the optimal solution for the current state-estimation problem, i.e. the problem in Eqn. 4.1 needs to be solved. The solution is denoted as $(\mathbf{x}^*, \mathbf{a}, \boldsymbol{\lambda}^*)$. Note that inactive, i.e. non-binding, inequality constraints are disregarded, and the active ones are incorporated as equality constraints. Then, the KKT conditions are stated for the optimal solution:

$$\nabla_{\mathbf{x}} J(\mathbf{x}^*, \mathbf{a}) + \sum_{k=1}^o \lambda_k^* \nabla_{\mathbf{x}} c_k(\mathbf{x}^*, \mathbf{a}) + \sum_{j=1}^l \mu_j^* \nabla_{\mathbf{x}} g_j(\mathbf{x}^*, \mathbf{a}) = \mathbf{0} \quad (4.6)$$

$$c_k(\mathbf{x}^*, \mathbf{a}) = 0, k = 1, 2, \dots, o, \quad (4.7)$$

$$g_j(\mathbf{x}^*, \mathbf{a}) \leq 0, j = 1, 2, \dots, l \quad (4.8)$$

$$\mu_j^* g_j(\mathbf{x}^*, \mathbf{a}) \leq 0, j = 1, 2, \dots, l \quad (4.9)$$

$$\mu_j^* \geq 0, j = 1, 2, \dots, l \quad (4.10)$$

where $\nabla_{\mathbf{x}}$ represents the gradient with respect to vector $\mathbf{x}$, and l and r are the numbers of equality and inequality constraints, respectively. The conditions in Eqn. 4.7-4.8 are the primal feasibility conditions, the conditions in Eqn. 4.9 are the complementary slackness conditions, and the conditions in Eqn. 4.10 impose the nonnegativity of the multipliers of the inequality constraints and are referred to as the dual feasibility conditions.

As the sensitivity analysis needs information about the dual problem, the required step before the sensitivity study is to obtain the dual variables solving the linear system of equations that is obtained from Eqns. 4.6-4.10:

$$\begin{bmatrix} \frac{\partial J}{\partial x_1} \\ \frac{\partial J}{\partial x_2} \\ \vdots \\ \frac{\partial J}{\partial x_n} \end{bmatrix} = \begin{bmatrix} \frac{\partial c_1}{\partial x_1} & \frac{\partial c_2}{\partial x_1} & \cdots & \frac{\partial c_o}{\partial x_1} \\ \frac{\partial c_1}{\partial x_2} & \frac{\partial c_2}{\partial x_2} & \cdots & \frac{\partial c_o}{\partial x_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial c_1}{\partial x_n} & \frac{\partial c_2}{\partial x_n} & \cdots & \frac{\partial c_o}{\partial x_n} \end{bmatrix} \cdot \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_o \end{bmatrix}. \quad (4.11)$$

If the gradient vectors of the binding constraints at a solution are linearly independent, the solution of the obtained system is guaranteed to be unique.

In the next step, the set of variables $\mathbf{x}^*$, $\mathbf{a}$, $\boldsymbol{\lambda}^*$, $\boldsymbol{\mu}^*$, J^* is perturbed in a way that the KKT conditions still hold [63]. Thus, to obtain the sensitivity equations, Eqn. 4.1 and Eqns. 4.6-4.10 are differentiated as follows:

$$[\nabla_{\mathbf{x}}J(\mathbf{x}^*, \mathbf{a})]^T d\mathbf{x} + [\nabla_{\mathbf{a}}J(\mathbf{x}^*, \mathbf{a})]^T d\mathbf{a} - dJ = 0 \quad (4.12)$$

$$\begin{aligned} & \left[\nabla_{\mathbf{xx}}J(\mathbf{x}^*, \mathbf{a}) + \sum_{k=1}^o \lambda_k^* \nabla_{\mathbf{xx}}c_k(\mathbf{x}^*, \mathbf{a}) + \sum_{j=1}^l \mu_j^* \nabla_{\mathbf{xx}}g_j(\mathbf{x}^*, \mathbf{a}) \right] d\mathbf{x} \\ & + \left[\nabla_{\mathbf{xa}}J(\mathbf{x}^*, \mathbf{a}) + \sum_{k=1}^o \lambda_k^* \nabla_{\mathbf{xa}}c_k(\mathbf{x}^*, \mathbf{a}) + \sum_{j=1}^l \mu_j^* \nabla_{\mathbf{xa}}g_j(\mathbf{x}^*, \mathbf{a}) \right] d\mathbf{a} \\ & + \nabla_{\mathbf{x}}\mathbf{c}(\mathbf{x}^*, \mathbf{a})d\boldsymbol{\lambda} + \nabla_{\mathbf{x}}\mathbf{g}(\mathbf{x}^*, \mathbf{a})d\boldsymbol{\mu} = \mathbf{0} \end{aligned} \quad (4.13)$$

$$[\nabla_{\mathbf{x}}\mathbf{c}(\mathbf{x}^*, \mathbf{a})]^T d\mathbf{x} + [\nabla_{\mathbf{a}}\mathbf{c}(\mathbf{x}^*, \mathbf{a})]^T d\mathbf{a} = \mathbf{0} \quad (4.14)$$

$$[\nabla_{\mathbf{x}}g_j(\mathbf{x}^*, \mathbf{a})]^T d\mathbf{x} + [\nabla_{\mathbf{a}}g_j(\mathbf{x}^*, \mathbf{a})]^T d\mathbf{a} = 0 \quad (4.15)$$

$$\text{if } \mu_j^* \neq 0, j \in \Omega_I$$

where $\nabla_{\mathbf{a}}$ represents the gradient with respect to vector $\mathbf{a}$, $\nabla_{\mathbf{xx}}$ and $\nabla_{\mathbf{xa}}$ represent Hessian with respect to $\mathbf{x}$, $\mathbf{x}$ and $\mathbf{x}$, $\mathbf{a}$ vectors, respectively. Ω_I is a set of active inequality constraints, and all the matrices are evaluated at the optimal solution $\mathbf{x}^*$, $\boldsymbol{\lambda}^*$, $\boldsymbol{\mu}^*$, J^* . At the solution of the estimation problem, binding inequality constraints are considered as equality constraints, and nonbinding ones are disregarded. It turns out that inequality constraints generally do not have influence in the solution of the state estimation problems, since equality constraints tightly condition that solution.

The system of equations is then formed as follows:

$$\begin{bmatrix} \mathbf{F}_{\mathbf{x}} & \mathbf{F}_{\mathbf{a}} & \mathbf{0} & -1 \\ \mathbf{F}_{\mathbf{xx}} & \mathbf{F}_{\mathbf{xa}} & \mathbf{C}_{\mathbf{x}}^T & 0 \\ \mathbf{C}_{\mathbf{x}} & \mathbf{C}_{\mathbf{a}} & \mathbf{0} & 0 \end{bmatrix} \cdot \begin{bmatrix} d\mathbf{x} \\ d\mathbf{a} \\ d\boldsymbol{\lambda} \\ dJ \end{bmatrix} = \mathbf{0} \quad (4.16)$$

where the vectors and submatrices (including their dimensions) are:

$$\mathbf{F}_{\mathbf{x}(1 \times n)} = [\nabla_{\mathbf{x}}J(\mathbf{x}^*, \mathbf{a})]^T, \quad (4.17)$$

$$\mathbf{F}_{\mathbf{a}(1 \times p)} = [\nabla_{\mathbf{a}}J(\mathbf{x}^*, \mathbf{a})]^T, \quad (4.18)$$

$$\mathbf{F}_{\mathbf{xx}(n \times n)} = \nabla_{\mathbf{xx}}J(\mathbf{x}^*, \mathbf{a}) + \sum_{k=1}^o \lambda_k^* \nabla_{\mathbf{xx}}c_k(\mathbf{x}^*, \mathbf{a}), \quad (4.19)$$

$$\mathbf{F}_{\mathbf{xa}(n \times p)} = \nabla_{\mathbf{xa}}J(\mathbf{x}^*, \mathbf{a}) + \sum_{k=1}^o \lambda_k^* \nabla_{\mathbf{xa}}c_k(\mathbf{x}^*, \mathbf{a}), \quad (4.20)$$

$$\mathbf{C}_{\mathbf{x}(o \times n)} = [\nabla_{\mathbf{x}}\mathbf{c}(\mathbf{x}^*, \mathbf{a})]^T, \quad (4.21)$$

$$\mathbf{C}_{\mathbf{a}(o \times p)} = [\nabla_{\mathbf{a}}\mathbf{c}(\mathbf{x}^*, \mathbf{a})]^T, \quad (4.22)$$

where p is the dimension of vector $\mathbf{a}$.

To calculate the sensitivities of the state vector with respect to the data vector, the system in Eqn. 4.16 is reformulated as:

$$\mathbf{U} \begin{bmatrix} d\mathbf{x} & d\boldsymbol{\lambda} & dJ \end{bmatrix}^T = \mathbf{S}d\mathbf{a} \quad (4.23)$$

Replacing $d\mathbf{a}$ by the p -dimensional identity matrix $\mathbf{I}$, the matrices $\mathbf{U}$ and $\mathbf{S}$ become:

$$\mathbf{U} = \begin{bmatrix} \mathbf{F}_x & \mathbf{0} & -1 \\ \mathbf{F}_{xx} & \mathbf{C}_x^T & 0 \\ \mathbf{C}_x & \mathbf{0} & 0 \end{bmatrix} \quad (4.24)$$

$$\mathbf{S}^T = -[\mathbf{F}_a \quad \mathbf{F}_{xa} \quad \mathbf{C}_a]$$

from which it follows that:

$$\begin{bmatrix} \frac{\partial \mathbf{x}}{\partial \mathbf{a}} & \frac{\partial \boldsymbol{\lambda}}{\partial \mathbf{a}} & \frac{\partial J}{\partial \mathbf{a}} \end{bmatrix}^T = \mathbf{U}^{-1} \mathbf{S}. \quad (4.25)$$

In Eqn. 4.25 sensitivities of the state variables, Lagrange multipliers, and the objective function are expressed with respect to the data vector. Eqn. 4.25 can be solved as follows:

$$\begin{bmatrix} \frac{\partial \mathbf{x}}{\partial \mathbf{a}} & \frac{\partial \boldsymbol{\lambda}}{\partial \mathbf{a}} \end{bmatrix}^T = -\mathbf{H}_x^{-1} \mathbf{H}_a \quad (4.26)$$

$$\frac{\partial J}{\partial \mathbf{a}} = \mathbf{F}_a + \mathbf{F}_x \frac{\partial \mathbf{x}}{\partial \mathbf{a}} \quad (4.27)$$

where matrices $\mathbf{H}_x$ and $\mathbf{H}_a$ are:

$$\mathbf{H}_x = \begin{bmatrix} \mathbf{F}_{xx} & \mathbf{C}_x^T \\ \mathbf{C}_x & \mathbf{0} \end{bmatrix} \quad (4.28)$$

$$\mathbf{H}_a = \begin{bmatrix} \mathbf{F}_{xa} \\ \mathbf{C}_a \end{bmatrix}.$$

If the power system in consideration is observable, then matrices $\mathbf{U}$ and $\mathbf{H}_x$ are invertible. Since distribution networks have radial or very weakly meshed topologies, matrices $\mathbf{U}$ and $\mathbf{H}_x$ are highly sparse, thus, they are easily factorized using sparse-oriented algorithms.

The solution to the system in Eqn. 4.25 yields sensitivity expressions with respect to the data vector $\mathbf{a}$, which contains measurements and system-admittance elements. From an engineering perspective, the important parameters are the sensitivities with respect to the conductor lengths, i.e. their impedances (since the conductor resistances and reactances are linearly dependent on the conductor's length [40]). The chain rule must be used to obtain the sensitivities with respect to the line impedances from sensitivities with respect to the admittance elements. The partial derivatives for a state variable with respect to conductor resistance and reactance are:

$$\begin{aligned} \frac{\partial x}{\partial r_{ij}^{mn}} &= \frac{\partial G_{ij}^{mn}}{\partial r_{ij}^{mn}} \left(\frac{\partial x}{\partial G_{ij}^{mn}} + \frac{\partial x}{\partial G_{ji}^{mn}} - \frac{\partial x}{\partial G_{ii}^{mn}} - \frac{\partial x}{\partial G_{jj}^{mn}} \right) \\ &+ \frac{\partial B_{ij}^{mn}}{\partial r_{ij}^{mn}} \left(\frac{\partial x}{\partial B_{ij}^{mn}} + \frac{\partial x}{\partial B_{ji}^{mn}} - \frac{\partial x}{\partial B_{ii}^{mn}} - \frac{\partial x}{\partial B_{jj}^{mn}} \right) \end{aligned} \quad (4.29)$$

$$\begin{aligned} \frac{\partial x}{\partial x_{ij}^{mn}} &= \frac{\partial G_{ij}^{mn}}{\partial x_{ij}^{mn}} \left(\frac{\partial x}{\partial G_{ij}^{mn}} + \frac{\partial x}{\partial G_{ji}^{mn}} - \frac{\partial x}{\partial G_{ii}^{mn}} - \frac{\partial x}{\partial G_{jj}^{mn}} \right) \\ &+ \frac{\partial B_{ij}^{mn}}{\partial x_{ij}^{mn}} \left(\frac{\partial x}{\partial B_{ij}^{mn}} + \frac{\partial x}{\partial B_{ji}^{mn}} - \frac{\partial x}{\partial B_{ii}^{mn}} - \frac{\partial x}{\partial B_{jj}^{mn}} \right), \end{aligned} \quad (4.30)$$

where partial derivatives with respect to system admittance elements are known from Eqn. 4.25, and the admittance element partial derivatives are:

$$\frac{\partial G_{ij}^{mn}}{\partial r_{ij}^{mn}} = \frac{\partial B_{ij}^{mn}}{\partial x_{ij}^{mn}} = \frac{(r_{ij}^{mn})^2 - (x_{ij}^{mn})^2}{((r_{ij}^{mn})^2 + (x_{ij}^{mn})^2)^2} \quad (4.31)$$

$$\frac{\partial G_{ij}^{mn}}{\partial x_{ij}^{mn}} = -\frac{\partial B_{ij}^{mn}}{\partial r_{ij}^{mn}} = \frac{2r_{ij}^{mn}x_{ij}^{mn}}{((r_{ij}^{mn})^2 + (x_{ij}^{mn})^2)^2}. \quad (4.32)$$

Besides the sensitivities of the state variables, the sensitivities of the derived variables, such as power flows or power injections, are also of interest. The way to obtain them is, again, with the application of the chain rule. E.g. the sensitivity of power flow in phase a with respect to its self series resistance is:

$$\begin{aligned}
\frac{\partial P_{km}^a}{\partial g_{km}^{aa}} &= \frac{\partial P_{km}^a}{\partial g_{km}^{aa}} + \frac{\partial P_{km}^a}{\partial |V_k^a|} \frac{\partial |V_k^a|}{\partial g_{km}^{aa}} + \frac{\partial P_{km}^a}{\partial |V_k^b|} \frac{\partial |V_k^b|}{\partial g_{km}^{aa}} \\
&+ \frac{\partial P_{km}^a}{\partial |V_k^c|} \frac{\partial |V_k^c|}{\partial g_{km}^{aa}} + \frac{\partial P_{km}^a}{\partial |V_m^a|} \frac{\partial |V_m^a|}{\partial g_{km}^{aa}} + \frac{\partial P_{km}^a}{\partial |V_m^b|} \frac{\partial |V_m^b|}{\partial g_{km}^{aa}} \\
&+ \frac{\partial P_{km}^a}{\partial |V_m^c|} \frac{\partial |V_m^c|}{\partial g_{km}^{aa}} + \frac{\partial P_{km}^a}{\partial \phi_k^a} \frac{\partial \phi_k^a}{\partial g_{km}^{aa}} + \frac{\partial P_{km}^a}{\partial \phi_k^b} \frac{\partial \phi_k^b}{\partial g_{km}^{aa}} \\
&+ \frac{\partial P_{km}^a}{\partial \phi_k^c} \frac{\partial \phi_k^c}{\partial g_{km}^{aa}} + \frac{\partial P_{km}^a}{\partial \phi_m^a} \frac{\partial \phi_m^a}{\partial g_{km}^{aa}} + \frac{\partial P_{km}^a}{\partial \phi_m^b} \frac{\partial \phi_m^b}{\partial g_{km}^{aa}} \\
&+ \frac{\partial P_{km}^a}{\partial \phi_m^c} \frac{\partial \phi_m^c}{\partial g_{km}^{aa}}.
\end{aligned} \tag{4.33}$$

4.1.3 Implementation

The perturbation approach was implemented in the computer program using C++ language. All the derivatives were implemented numerically. LAV and SHGM estimators produce objective functions that have discontinuous derivatives. Since we are interested in small deviations of model and measurement parameters, we do not operate in the area of discontinuities with the SHGM estimator. This is not the case for the LAV estimator, where discontinuity is encountered at $r_i = 0$. For the LAV estimator, the problem is restated as:

$$J(\mathbf{x}, \mathbf{a}) = \sum_{i=1}^m r_i \tag{4.34}$$

and the measurements are imposed as constraints [65], i.e. $(f_i - h_i(\mathbf{x}))/\sigma_i \leq r_i$ and $(f_i - h_i(\mathbf{x}))/\sigma_i \geq -r_i$.

For numerical implementation, the following approximations were used:

$$\frac{\partial f(a, b)}{\partial x} \approx \frac{f(a + h, b) - f(a, b)}{h} \tag{4.35}$$

$$\frac{\partial^2 f(a, b)}{\partial^2 x} \approx \frac{-f(a - h, b) + 2f(a, b) - f(a + h, b)}{h^2} \tag{4.36}$$

$$\frac{\partial^2 f(a, b)}{\partial x \partial y} \approx \frac{f(a + h_1, b + h_2) - f(a + h_1, b - h_2) - f(a - h_1, b + h_2) + f(a - h_1, b - h_2)}{4h_1 h_2} \tag{4.37}$$

A formula for h that balances the rounding error against the secant error for optimum accuracy is:

$$h = 2\sqrt{\epsilon \left| \frac{f(x)}{f''(x)} \right|} \tag{4.38}$$

4.1.3.1 Implementation validation

The perturbation approach to the implementation of the sensitivity analysis was validated by comparing the obtained results with direct numerical derivatives. The measurements were generated from the load-flow results for the reference feeder loading provided by the

Table 4.1: Measurement configuration for validation.

Bus ID	PMU	PQ injection	Equality constr.
650			
632	•		•
633	•		•
634		•	
645	•		
646		•	
6320	•		
671	•		
692	•		
675		•	
680		•	•
684	•		•
652		•	
611		•	

IEEE [53], more precisely the validation was performed on the 13-bus IEEE distribution test feeder [53] shown in Fig. A.1.

The validation was made on a single configuration of measurements, which is listed in Table 4.1. The PMU column indicates the presence of a PMU measurement on a corresponding bus, the PQ injection bus indicates the presence of an active/reactive power-injection measurement on the corresponding bus, and the equality constr. column indicates whether zero injection constraints are imposed on the corresponding bus. For validation purposes, the measurements were assumed to be perfectly accurate without added noise. The direct numerical derivatives were computed as follows: first, the state estimation was computed to obtain the reference state vector $\mathbf{x}_{ref}$, then the parameter in question was perturbed $a + \Delta a$ and the state estimation was computed again with a perturbation to obtain the state vector $\mathbf{x}_p$, and the derivatives were then obtained as follows:

$$\frac{\partial \mathbf{x}}{\partial a} \approx \frac{\mathbf{x}_p - \mathbf{x}_{ref}}{\Delta a}. \quad (4.39)$$

The cases presented here are the phase-angle sensitivity across all buses with respect to the active power measurement on bus 634, and the magnitude sensitivity across all buses with respect to the admittance matrix element $\mathbf{Y}_{652a-684a}$. The results are shown in Fig. 4.1 and Fig. 4.2. The lower graphs represent the error with respect to the direct numerical derivative. It is clear that there is some difference between the perturbation approach results and the direct numerical derivatives. This difference is attributed to numerical errors since, in the perturbation approach, all the derivatives are computed numerically.

4.1.4 Error bound

The sensitivities obtained with the solution to the system in Eqn. 4.25 represent tangent lines at the optimal solution point. These derivatives are only truly valid for that point; however, from an engineering perspective, we are more interested in intervals, e.g. we would like to know within which interval the state vector can be expected if we are confident that the line length is within a certain range.

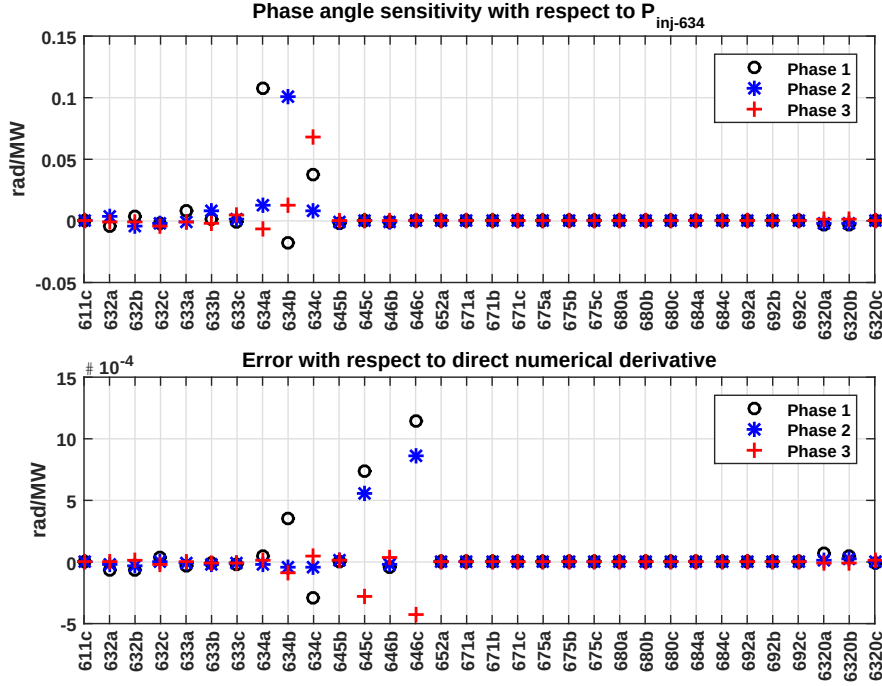


Figure 4.1: Sensitivity of phase angles with respect to the active power measurement on bus 634.

The interval $[-\mathbf{x}_a, \mathbf{x}_a]$ can be obtained from:

$$\pm \mathbf{x}_a \approx \mathbf{x}^* \pm \Delta \mathbf{x} = \mathbf{x}^* \pm \Delta a \frac{\partial \mathbf{x}}{\partial a}. \quad (4.40)$$

Since the derivative is only valid for the optimal solution, an error will invariably be made with such an interval calculation.

An example for the phase part of the state-vector error is shown in Fig. 4.3, for 684–652 conductor impedance ($z_{684a-652a}$). The error is computed as:

$$E = \max \left| \mathbf{x}_{true} - \left(\mathbf{x}^* + \Delta z_{684a-652a} \frac{\partial \mathbf{x}^*}{\partial z_{684a-652a}} \right) \right| \quad (4.41)$$

The conductor has a specified length of 800 feet, while the computation was performed for an impedance interval corresponding to a conductor length in the range from 600 to 1000 feet ($\pm 25\%$). It is clear that though the error is small it quickly increases as we move away from the value at the optimal solution $\mathbf{x}^*$. Thus, a bound that would specify the maximum state vector error when a parameter interval analysis is performed is of interest. The error bound for the linear approximation would normally require knowledge of the second-order derivatives $\frac{\partial^2 \mathbf{x}}{\partial \mathbf{a}^2}$. Calculating them is impracticable for the problem at hand.

Another approach is to take advantage of the nature of the problem and the radial topology of distribution systems. The only two things that influence the state variables are line impedances and feeder loading. In radial networks, the largest decrease of the state variables (voltage magnitudes and phase angles) is achieved when the estimated feeder loading is the largest and when the given line impedances are the greatest (and vice-versa). To illustrate this, simulations were performed for six different feeder loadings and line impedances. For each run, the loads as well as line impedances were increased.

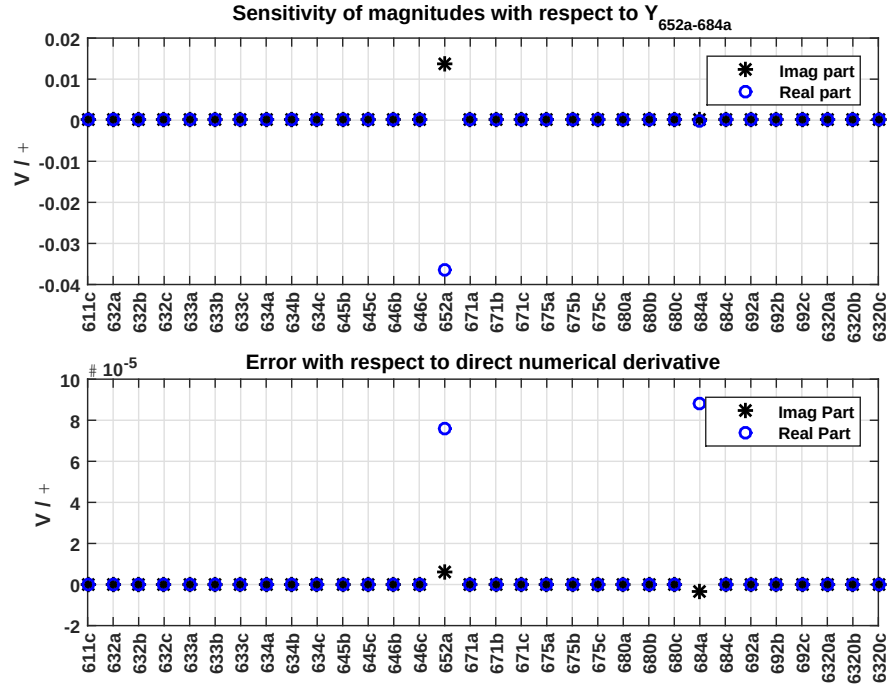


Figure 4.2: Sensitivity of voltage magnitudes with respect to the admittance matrix element.

Figs. 4.4 and 4.5 depict that voltage magnitudes and voltage phases decrease on all the buses for each consecutive run.

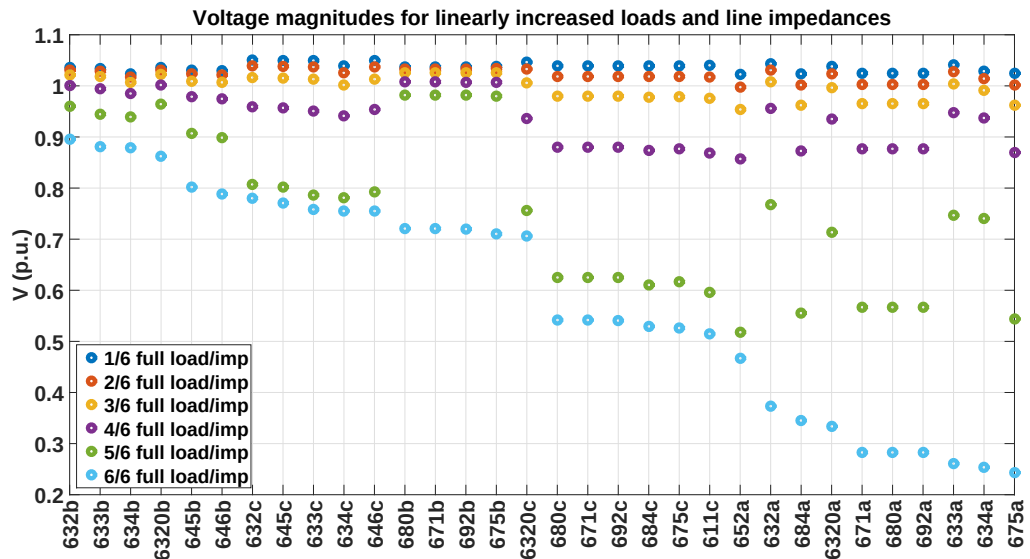


Figure 4.4: Voltage magnitudes for linearly increased loads and impedances.

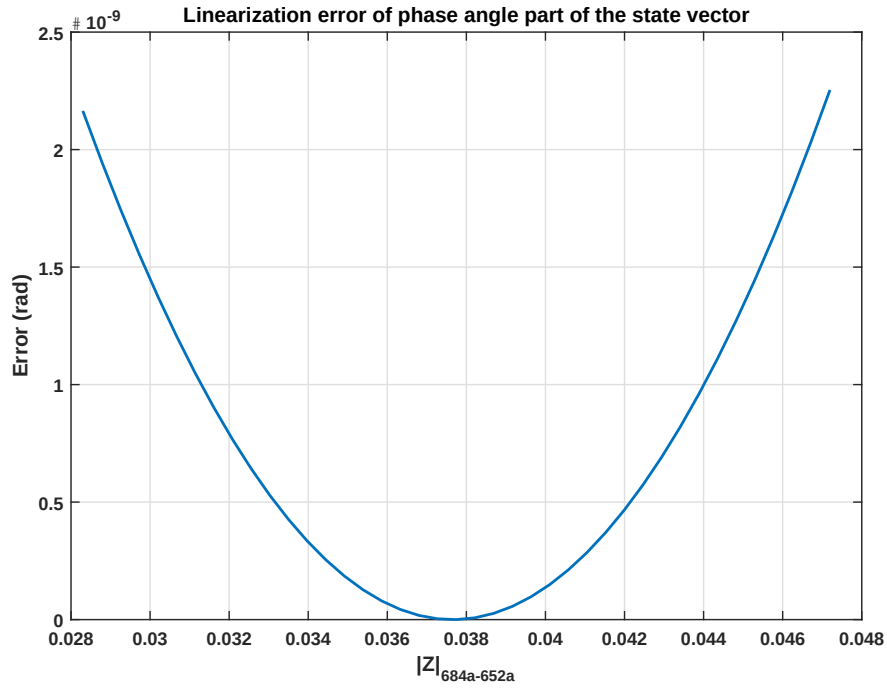


Figure 4.3: Linearization error of phase-angle part of the state vector.

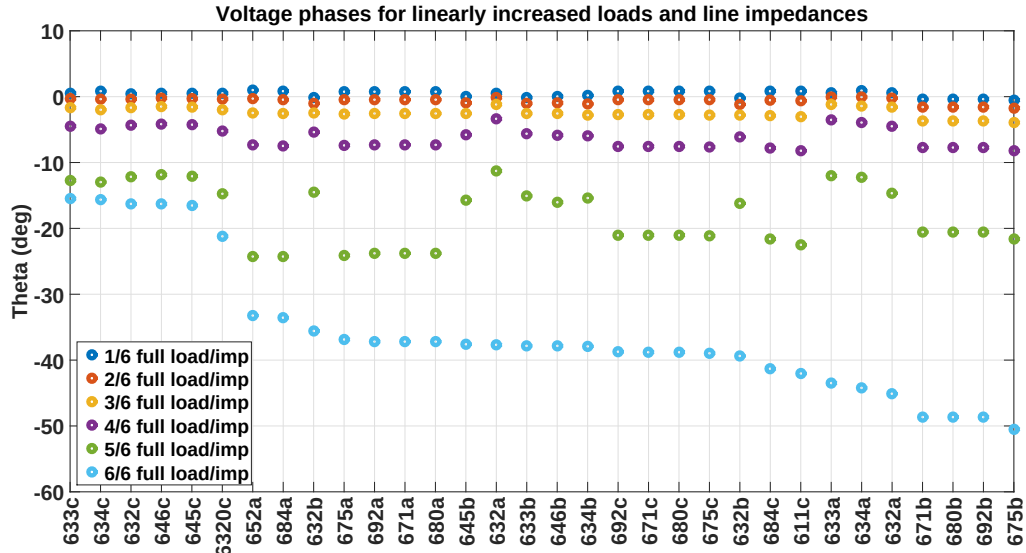


Figure 4.5: Voltage phases for linearly increased loads and impedances.

From this the rules, for data vectors $\mathbf{a}_u$ and $\mathbf{a}_l$ that will yield the state vectors with the largest deviation from $\mathbf{x}^*$ in both directions (i.e., $\mathbf{x}_u$ and $\mathbf{x}_l$) can be inferred. For power-injection and flow measurements, the upper and lower interval bounds are taken for

$\mathbf{a}_u$ and $\mathbf{a}_l$, respectively:

$$\begin{aligned} \{\mathbf{P}_{i,u}, \mathbf{Q}_{i,u}, \mathbf{P}_{ij,u}, \mathbf{Q}_{ij,u}\} &= \{\mathbf{P}_i, \mathbf{Q}_i, \mathbf{P}_{ij}, \mathbf{Q}_{ij}\} \\ &+ \{\Delta\mathbf{P}_i, \Delta\mathbf{Q}_i, \Delta\mathbf{P}_{ij}, \Delta\mathbf{Q}_{ij}\} \end{aligned} \quad (4.42)$$

$$\begin{aligned} \{\mathbf{P}_{i,l}, \mathbf{Q}_{i,l}, \mathbf{P}_{ij,l}, \mathbf{Q}_{ij,l}\} &= \{\mathbf{P}_i, \mathbf{Q}_i, \mathbf{P}_{ij}, \mathbf{Q}_{ij}\} \\ &- \{\Delta\mathbf{P}_i, \Delta\mathbf{Q}_i, \Delta\mathbf{P}_{ij}, \Delta\mathbf{Q}_{ij}\} \end{aligned} \quad (4.43)$$

For line resistances and reactances, the upper interval bounds are also used:

$$\{\mathbf{r}_{ij,u}, \mathbf{x}_{ij,u}\} = \{\mathbf{r}_{ij}, \mathbf{x}_{ij}\} + \{\Delta\mathbf{x}_{ij}, \Delta\mathbf{r}_{ij}\} \quad (4.44)$$

$$\{\mathbf{r}_{ij,l}, \mathbf{x}_{ij,l}\} = \{\mathbf{r}_{ij}, \mathbf{x}_{ij}\} - \{\Delta\mathbf{x}_{ij}, \Delta\mathbf{r}_{ij}\} \quad (4.45)$$

For voltage magnitudes, the lower interval bounds are used:

$$|\mathbf{V}|_{i,u} = |\mathbf{V}|_i - \Delta|\mathbf{V}|_i \quad (4.46)$$

$$|\mathbf{V}|_{i,l} = |\mathbf{V}|_i + \Delta|\mathbf{V}|_i \quad (4.47)$$

For voltage phase angles, the lower interval bounds are also used:

$$\Theta_{i,u} = \Theta_i - \Delta\Theta_i \quad (4.48)$$

$$\Theta_{i,l} = \Theta_i + \Delta\Theta_i \quad (4.49)$$

Data vectors are, thus, formed as:

$$\mathbf{a}_u = [\mathbf{P}_{i,u}, \mathbf{Q}_{i,u}, \mathbf{P}_{ij,u}, \mathbf{Q}_{ij,u}, \mathbf{r}_{ij,u}, \mathbf{x}_{ij,u}, |\mathbf{V}|_{i,u}, \Theta_{i,u}]. \quad (4.50)$$

$$\mathbf{a}_l = [\mathbf{P}_{i,l}, \mathbf{Q}_{i,l}, \mathbf{P}_{ij,l}, \mathbf{Q}_{ij,l}, \mathbf{r}_{ij,l}, \mathbf{x}_{ij,l}, |\mathbf{V}|_{i,l}, \Theta_{i,l}]. \quad (4.51)$$

Using them, the state vectors $\mathbf{x}_u$ and $\mathbf{x}_l$ are obtained. With the obtained state vectors, the upper error bounds on the interval calculations can be specified as:

$$\left| \mathbf{x}_u - \left(\mathbf{x}^* + \Delta a \frac{\partial \mathbf{x}^*}{\partial a} \right) \right| \geq \left| \mathbf{x}_{true} - \left(\mathbf{x}^* + \Delta a \frac{\partial \mathbf{x}^*}{\partial a} \right) \right| \quad (4.52)$$

$\forall a \in [\mathbf{a}, \mathbf{a}_u]$

$$\left| \mathbf{x}_l - \left(\mathbf{x}^* + \Delta a \frac{\partial \mathbf{x}^*}{\partial a} \right) \right| \geq \left| \mathbf{x}_{true} - \left(\mathbf{x}^* + \Delta a \frac{\partial \mathbf{x}^*}{\partial a} \right) \right| \quad (4.53)$$

$\forall a \in [\mathbf{a}, \mathbf{a}_l]$

These error bounds only apply to radial networks.

4.1.5 Results

The sensitivity calculations were performed for the three different measurement configurations that are listed in Table 4.2, and for three different estimators, namely WLS, LAV, and SHGM. For each configuration, the sensitivities of the state variables with respect to the measurements and the conductor impedances (self and mutual) were of interest. Different measurement configurations and estimators yield a large number of possible combinations, and although we conducted an exhaustive number of simulations, we choose to depict the most representative results. The following subsections provide the analysis, simulations, and results for the uncertain conductor length, and the uncertain measurements.

Table 4.2: Measurement configuration for calculation of sensitivities.

Configuration	PQ		PQ-PMU		PMU-PQ	
Bus ID	PQ	PMU	PQ	PMU	PQ	PMU
632	•		•	•	•	•
633	•		•	•	•	•
634	•		•	•		•
645	•		•	•	•	•
646	•		•	•		•
6320	•		•	•		•
671	•		•	•		•
692	•		•	•		•
675	•		•	•	•	•
680	•		•	•	•	•
684	•		•	•		•
652	•		•	•	•	•
611	•		•	•	•	•

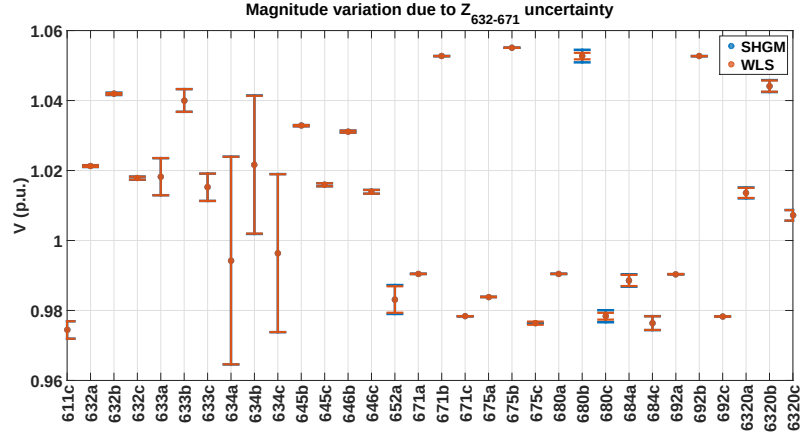
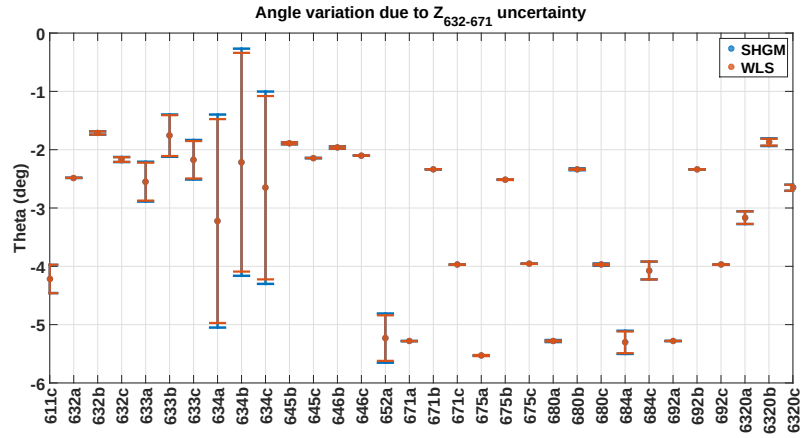
4.1.6 Variation of the line length

In practice, we often encounter differences between modeled line lengths and the actual line lengths. Normally, the modeled line lengths are based on the geographical routes not based on the actual lengths of the laid cable. The actual route has many small detours from the modeled route and those detours can add up to significant percentage of the total cable length. This problem is a bit less severe but still present in overhead lines. Here, the sags between poles are not accounted for.

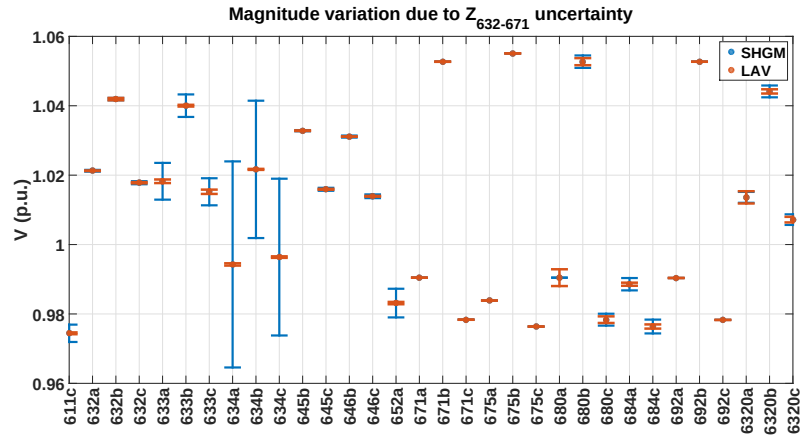
The line between buses 6320 and 671 is 1500 feet long. A $\pm 20\%$ interval accounts for ± 300 feet. Since the line has a three-phase configuration, the line impedance matrix has three self-impedances and three mutual-impedances. The following subsections depict the results for different measurement configurations.

4.1.6.1 Measurement configuration PQ

Figs. 4.6 and 4.7 show the interval of the magnitude and phase parts of the state vector, respectively, with respect to the $z_{6320-671}$, which represents uncertain line length, for the SHGM and WLS estimators. It is seen that the local sensitivities of the two estimators are almost identical.

Figure 4.6: Interval of the magnitude part of the state vector due to uncertain $\mathbf{Z}_{6320-671}$.Figure 4.7: Interval of the angle part of the state vector due to uncertain $\mathbf{Z}_{6320-671}$.

Figs. 4.8 and 4.9 show the local sensitivity for the LAV estimator with comparison to WLS. It is clear that the influence of uncertain line length is much smaller when the LAV estimator is used.

Figure 4.8: Interval of the magnitude part of the state vector due to uncertain $\mathbf{Z}_{6320-671}$.

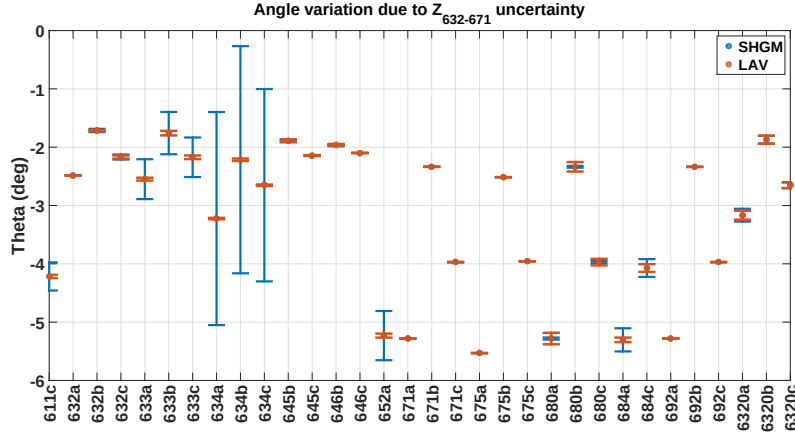


Figure 4.9: Interval of the angle part of the state vector due to uncertain $\mathbf{Z}_{6320-671}$.

Further, the uncertainty interval is the largest on buses 633 and 634. The probable reason for this is that the series impedance of the transformer on branch 633 – 634 is two orders of magnitude higher than the series impedance of any other branch. The uncertainty spreads across all the buses due to the measurement configuration and the nature of power injection measurement functions (Eqn. 2.32 and Eqn. 2.33).

4.1.6.2 Measurement configuration PQ-PMU

In this measurement configuration, the effect of the uncertain line length on the estimated state variables is much smaller for the WLS and SHGM estimators and remains negligible for the LAV estimator as Figs. 4.10 and 4.11 show.

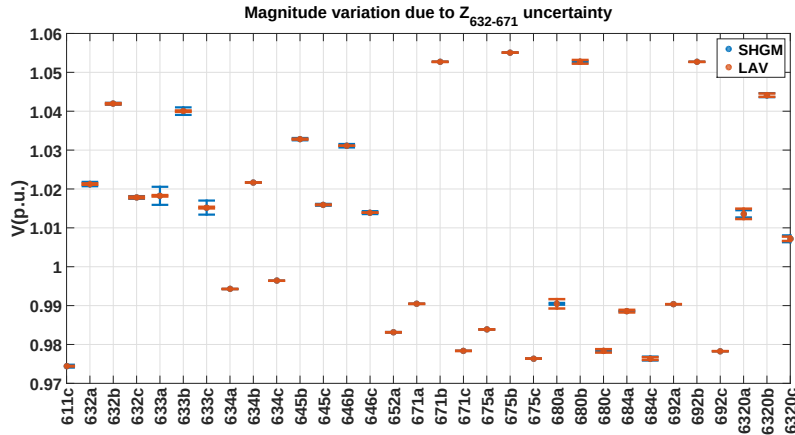
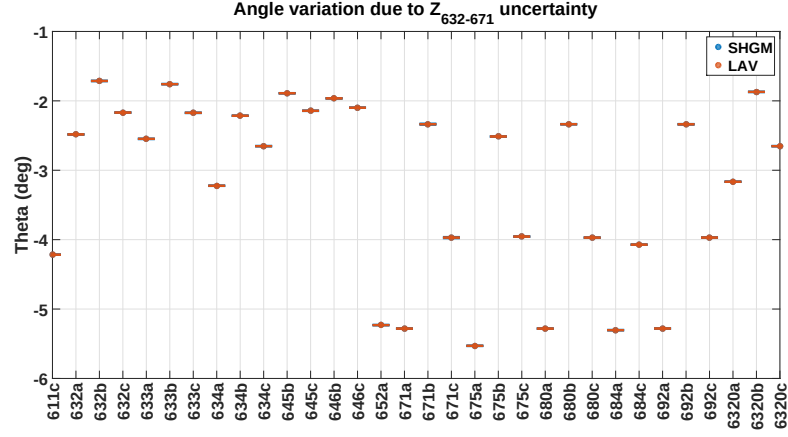


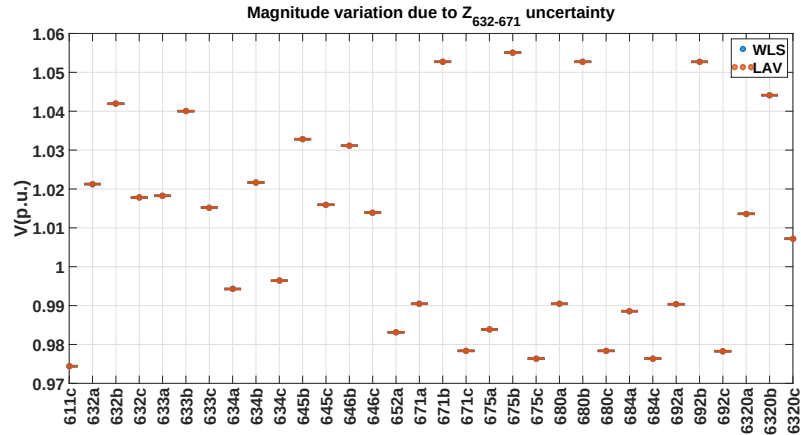
Figure 4.10: Interval of the magnitude part of the state vector due to uncertain $\mathbf{Z}_{6320-671}$.

Figure 4.11: Interval of the angle part of the state vector due to uncertain $\mathbf{Z}_{6320-671}$.

4.1.6.3 Measurement configuration PMU-PQ

The effect of the uncertain line length on the state vector in this configuration is even smaller than that in the previous two configurations, for all four estimators. Figs. 4.12 and 4.13 depict the results for WLS and LAV estimators. The two are virtually indistinguishable. We omitted the results for SHGM, but the results are no different for this estimator.

The reason for such small influence can be found in the measurement configuration in which PMUs represent the majority of devices. Phase-angle and magnitude measurements alone form a linear state-estimation problem, where topology has no effect on the state variables. With the introduction of additional power-injection measurements, the topology has an effect on certain state variables, but since the measurements are only placed on a few buses and the influence of topology uncertainty spreads in accordance with Eqn. 2.82 and Eqn. 2.83, the majority of buses remain isolated from the uncertainty influence.

Figure 4.12: Interval of the magnitude part of the state vector due to uncertain $\mathbf{Z}_{6320-671}$.

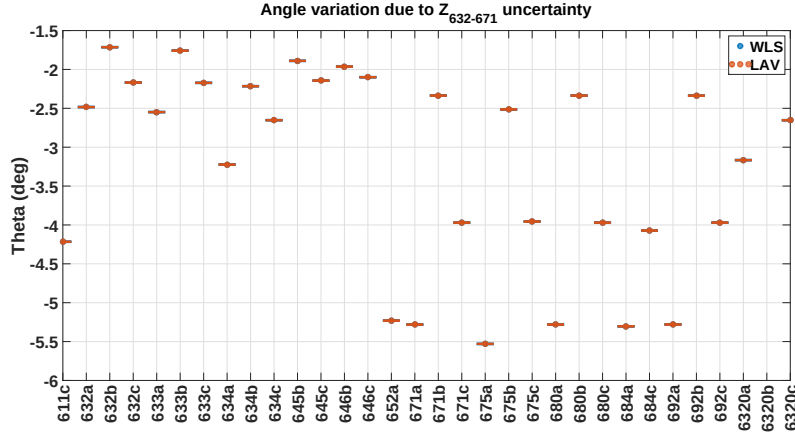


Figure 4.13: Interval of the angle part of the state vector due to uncertain $Z_{632-671}$.

4.1.7 Quality of measurements

Measurement devices are susceptible to several types of errors. The first type of measurement errors are small drifts of measured value compared to the true value. For example, time drifts of certain analog elements can cause errors in the measurement. The second type are gross measurement errors e.g. a current transducer may start malfunctioning and the readings would be several orders of magnitude off the true value. In this study, we are interested in the first type of measurement errors, as the second type requires a different type of analysis.

For each of the measurement configurations and different estimators, the effect of uncertain measurements was computed. Different measurement weights determine the magnitude of the influence of a particular measurement in comparison to others, but to simplify the problem, the same weights were assigned to all measurements.

4.1.7.1 Measurement configuration PQ

The variation of the state vector for a $\pm 20\%$ uncertainty interval on power-injection measurements on bus 671 is shown in Fig. 4.14 and Fig. 4.15 for the WLS and SHGM estimators. It is clear that the estimators have a similar local sensitivity. Surprisingly, the LAV estimator has a much greater local sensitivity as is depicted in Figs. 4.16 and 4.17.

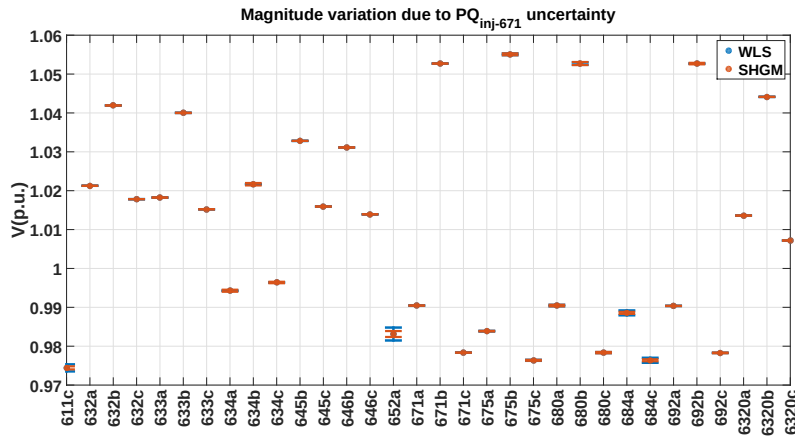
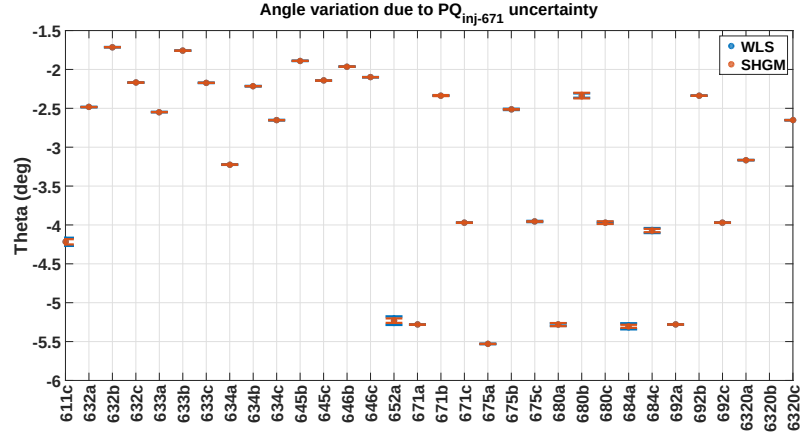
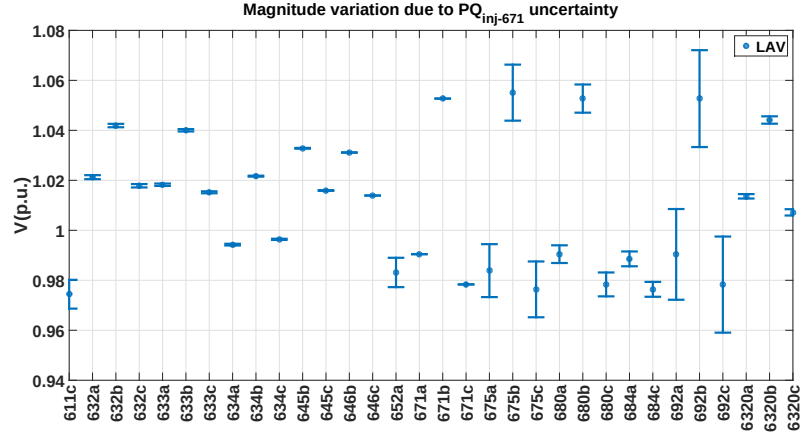
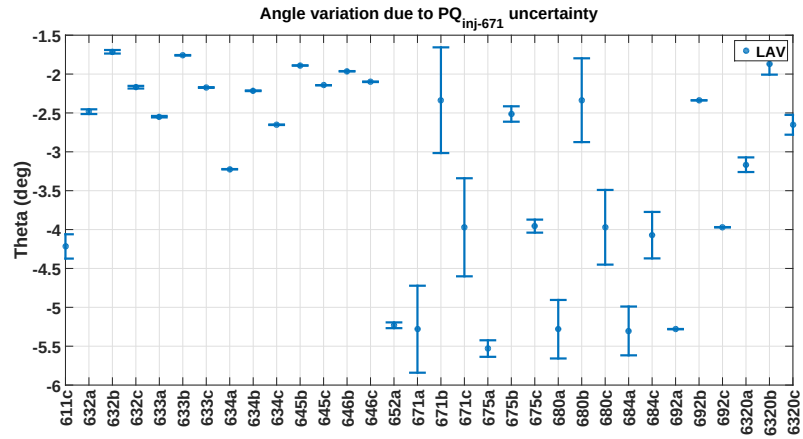


Figure 4.14: Interval of the magnitude part of the state vector due to uncertain $PQ_{inj-671}$.

Figure 4.15: Interval of the angle part of the state vector due to uncertain $PQ_{inj-671}$.Figure 4.16: Interval of the magnitude part of the state vector due to uncertain $PQ_{inj-671}$.Figure 4.17: Interval of the angle part of the state vector due to uncertain $PQ_{inj-671}$.

4.1.7.2 Measurement configuration PQ-PMU

The variation of the state vector for a $\pm 20\%$ uncertainty interval of the power injection measurement on bus 671 is shown in Figs. 4.18 4.19. It is clear that the angle-measurement variation is barely observable in both the LAV and SHGM estimators. Interestingly, it can be observed that, similarly to the PQ configuration, the LAV estimator has a greater local sensitivity on the magnitude part than SHGM. The WLS estimator behaves much like the SHGM estimator.

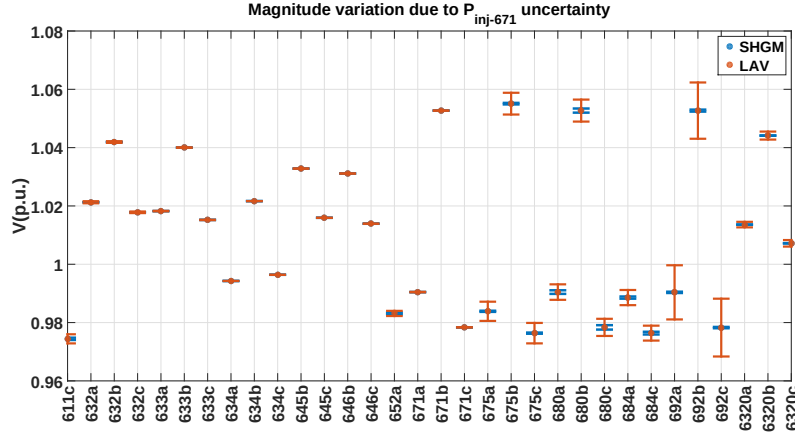


Figure 4.18: Interval of the magnitude part of the state vector due to uncertain $\mathbf{PQ}_{inj-671}$.

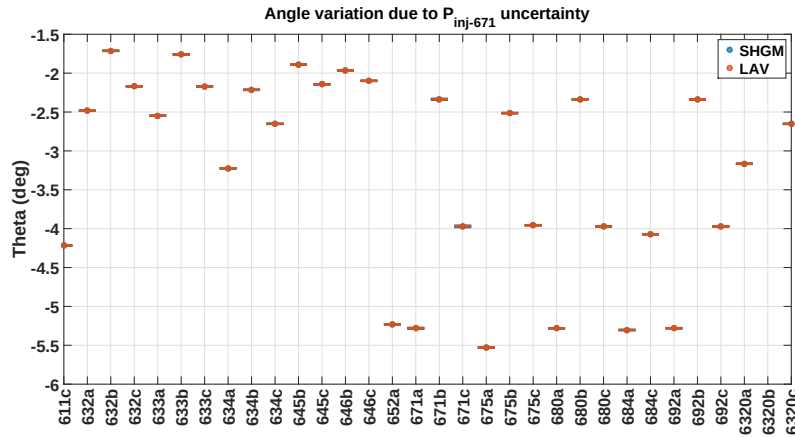
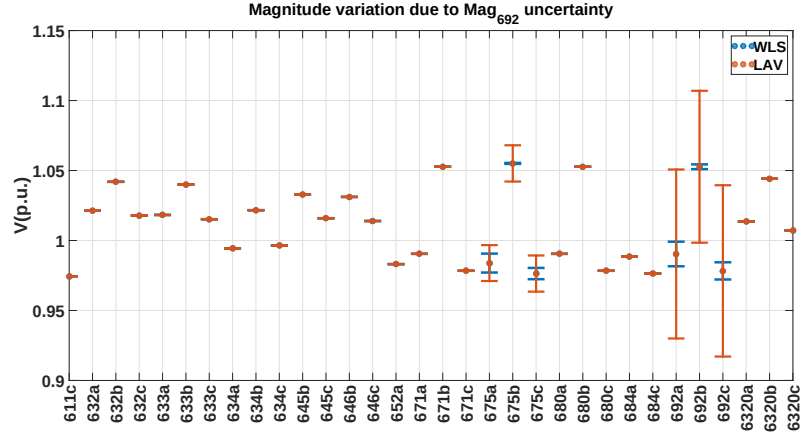
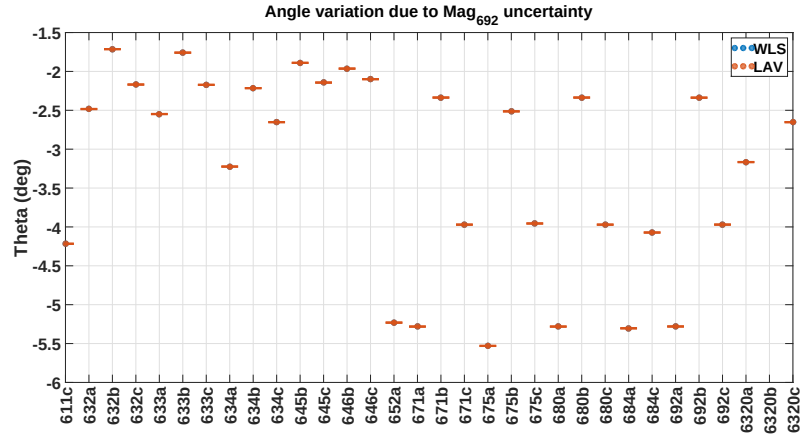


Figure 4.19: Interval of the angle part of the state vector due to uncertain $\mathbf{PQ}_{inj-671}$.

4.1.7.3 Measurement configuration PMU-PQ

Fig. 4.20 and Fig. 4.21 show the state vector uncertainty interval for $\pm 0.01^\circ$ uncertain phase measurements on bus 634. It is clear that the angle-measurement variation is almost fully translated into the state vector on buses 634 and 633, but remains isolated on the phase part of these two buses. Again, the reasons for this are to be found in the measurement configuration. The power injection measurement on bus 634 causes a spread of the uncertainty onto the adjacent bus.

Figure 4.20: Interval of the magnitude part of the state vector due to uncertain $|\mathbf{V}|_{692}$.Figure 4.21: Interval of the angle part of the state vector due to uncertain $|\mathbf{V}|_{692}$.

4.1.8 Summary

In this chapter, we investigated the local sensitivities of estimated state variables with respect to the uncertain line lengths and inaccurate (pseudo-) measurements in a three-phase distribution network for different measurement configurations and different estimators. We selected and implemented the approach with a perturbation of the KKT conditions, which is agnostic to the choice of the estimator. The analysis was performed for a full three-phase branch network model. The implemented estimators were LAV, WLS, and SHGM.

The results show that both the measurement configuration and the estimator choice have a large influence on the spread of the line-impedance and measurement uncertainties across the network buses in the estimated state vector. The LAV estimator has small local sensitivity with respect to line impedances in comparison to SHGM and WLS estimators. The opposite is the case for measurement sensitivities, where the results show that the LAV estimator has the worst performance among the implemented ones. It can be concluded that the LAV estimator is suitable for networks where line lengths are uncertain, but we have good information about measurement noise characteristics.

In terms of local sensitivities, the results show a very similar behavior for the SHGM and WLS estimators. However, among the two, only the SHGM estimator is considered robust in terms of gross measurement errors. Thus, the SHGM estimator based on projection

statistics is suggested in cases where line impedances are known with precision, but we are unsure about measurement noise characteristics.

Even though the influence of the line-impedance's uncertainty on the state-vector estimation is smaller for the LAV estimator and for measurement configurations with a prevailing number of PMU measurements (in comparison to those with the prevailing number of power-injection measurements), the line-impedances's uncertainty still influences the calculation of the derived variables (e.g., power flows). Thus, accurate knowledge of line-impedances is crucial for an accurate estimation of the state vector and its derived variables.

Chapter 5

Application Example

In this chapter, the application of the SE system to a section of the real-world network is described. To be functional, a SE system needs a number of components performing different functionalities architected into a solution that delivers the targeted performance. Performance is a parameter determined by the application needs. It is normally specified in the maximum error of the state estimates, and in the minimal frequency of the state estimates. These two parameters determine much of the system design factors, such as type of the measurement devices, configuration of measurements, computational and database resources, etc. The following sections and subsections describe the measurement pre-processing module, state estimation module, visualization and configuration module, and communication protocols between the separate software modules.

The SE system comprises one part of the complete measurement system. The other part is represented by the measurement devices that are deployed on the field. There are many options for measurement devices, as to the quantities they measure, accuracy, report rates, etc. Modern distribution networks will require accurate, real-time observation of their state, and to that end section 5.3 presents a design for a micro-synchrophasor measurement unit that is suitable for distribution networks. The device delivers highly accurate current and voltage phasor measurements at high time resolution. The description covers software and hardware design, but omits technical details.

5.1 Grid Section

The SE system was applied to the part of the Elektro Primorska (EP) distribution grid. The testbed contains 31 buses, 11 distribution transformers, 5 solar plants, and a co-generation. The single line diagram of the testbed is depicted in Fig. 5.1.

The modeling phase requires a thorough assesment of the system components. The following checklist was developed to help with the execution of this process:

- Determine the network topology. The current status in many DSO companies is that they do not posses a single unified repository with an updated topology, thus, the first task is to find the most up-to-date version of the topology.
- Determine the states of switches, circuit breakers, and disconnectors. It must be emphasized that the states must be determined as they exist in the field.
- A list of measurement devices already installed needs to be made. Their measurement capabilities, report rates, and accuracy need to be determined. Also, the access to measurements needs to be determined, and if any special permissions are necessary or any communication interfaces need to be developed.

- For overhead lines, the geometrical configuration as well as their material, diameter, and actual cross section need to be determined.
- For underground cables, the geometrical configuration and cable types need to be determined. It is best if the DSO company provides the cable datasheets that contain the parameters relevant to modeling.
- Connections of transformers are normally known, however distribution transformers normally have manual tap changers installed and normally their positions are not known, thus, they need to be determined in the field.
- Possible voltage regulators, the method of their compensation, and their settings are also important and need to be determined accurately.

In the next step, the need for any additional measurement devices needs to be assessed. If additional devices need to be installed, their placement needs to be determined. There are several heuristic algorithms that can determine their placement ([66], [67]). The placement methods usually only consider accuracy of the measurement devices and their influence on the statistical performance of the state estimation algorithm. Factors that also need to be considered are feasibility and cost of installation on a certain bus or branch. The cost of installation varies across buses and depends on things like existing equipment (e.g. voltage and current transformers), mounting location (pole, pad, or underground vault), cellular network coverage, etc. Since the cost of installation is comparable to the cost of the measurement device itself, it is sensible to install all measurement devices (types) on a determined location even if, for example, the placement algorithm determines only phasor measurements.

5.2 Software Architecture

In software architecture, we aimed for a loosely coupled architectural design that would allow an asynchronous communication style between function blocks, the dynamic binding of additional function blocks, and deployment of function blocks across different platforms.

The following list covers all the main function blocks that the solution needs to implement:

- *Measurement preprocessing*: measurements need to be preprocessed before they are used in the SE algorithm to sort them, detect possible gross errors, decimate or interpolate them, and assign weights.
- *State estimation solver*: is the implementation of the solver method of the selected state estimator.
- *Topology storage*: an up-to-date version of topology must be stored in a repository that is accessible to the state estimation solver.
- *Visualization, management, and diagnostics*: results must be represented to users in an organized and compelling way. The SE system also needs a central point of control and monitoring of all its functional parts.

Besides function blocks software architecture requires communication solutions for:

- *Communication between function blocks*: depending on the architectural design, a communication solution for sharing data between function blocks is needed.

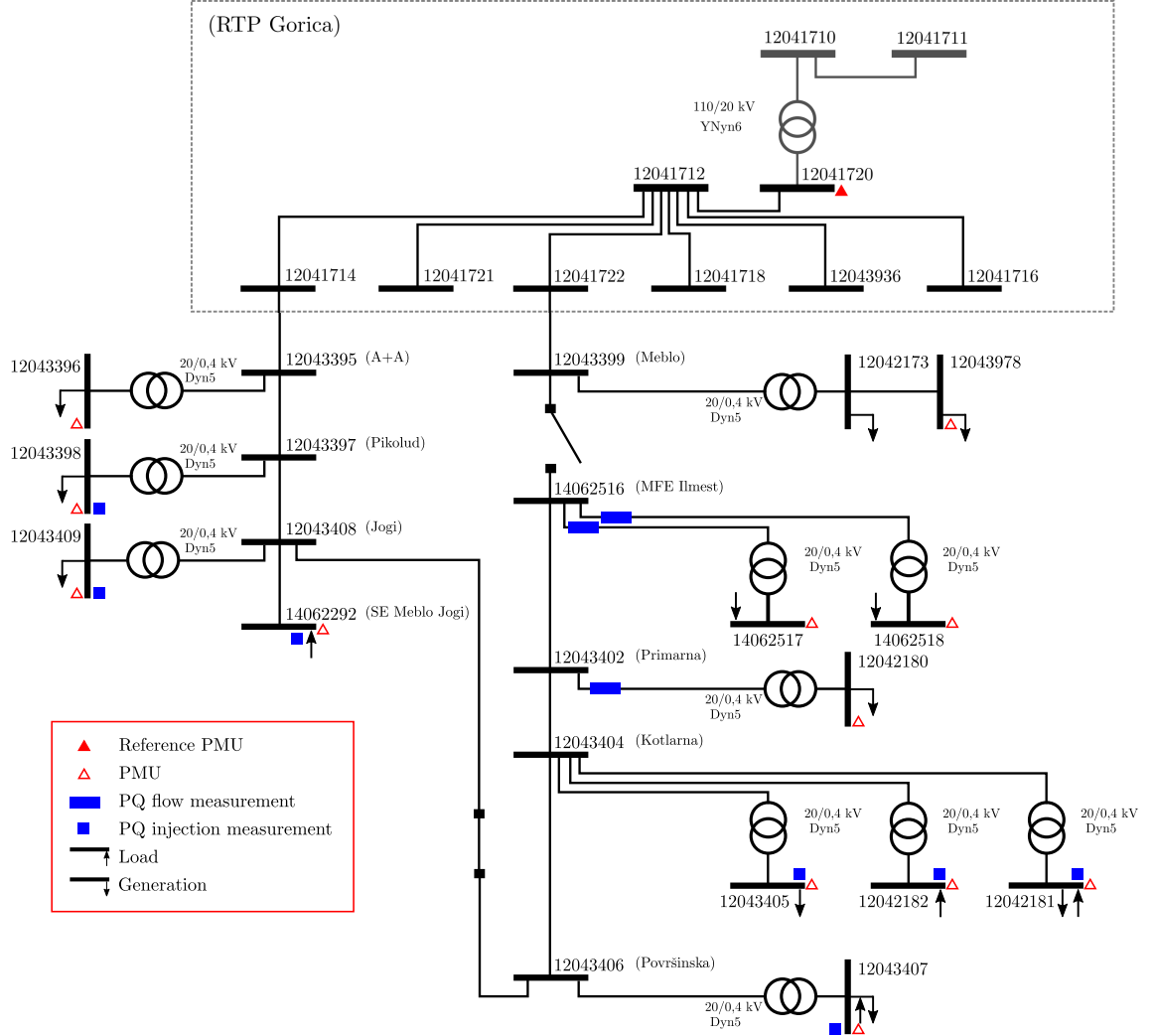


Figure 5.1: Single line diagram of the testbed.

- *Communication between the SE system and measurement devices:* measurement devices feature different communication interfaces, from power line communications (PLC) to different wireless solutions. The SE system needs a unified, communication solution-independent way of accessing the measurements.

The software architecture with the listed function blocks is depicted in Fig. 5.2. The following paragraphs present our solutions to each of the listed functions.

5.2.1 Measurement preprocessing

Measurements need to be preprocessed before they are used in the state estimation algorithm to detect possible gross errors, normalize them to the same time base using decimation or interpolation, assign measurement weights, and ensure their synchronization.

As the system is running, gross measurement errors can occur at one or more measurement points due to voltage or current sensor failure, or failure of the device measurement front-end. The state estimation algorithm itself can be designed to be statistically robust (e.g. the reviewed SHGM estimator) and can implicitly reject bad data. However if the selected state estimation algorithm is not robust (e.g. WLS estimator), the burden of bad data detection must be transferred to another function block in the software solution. Since

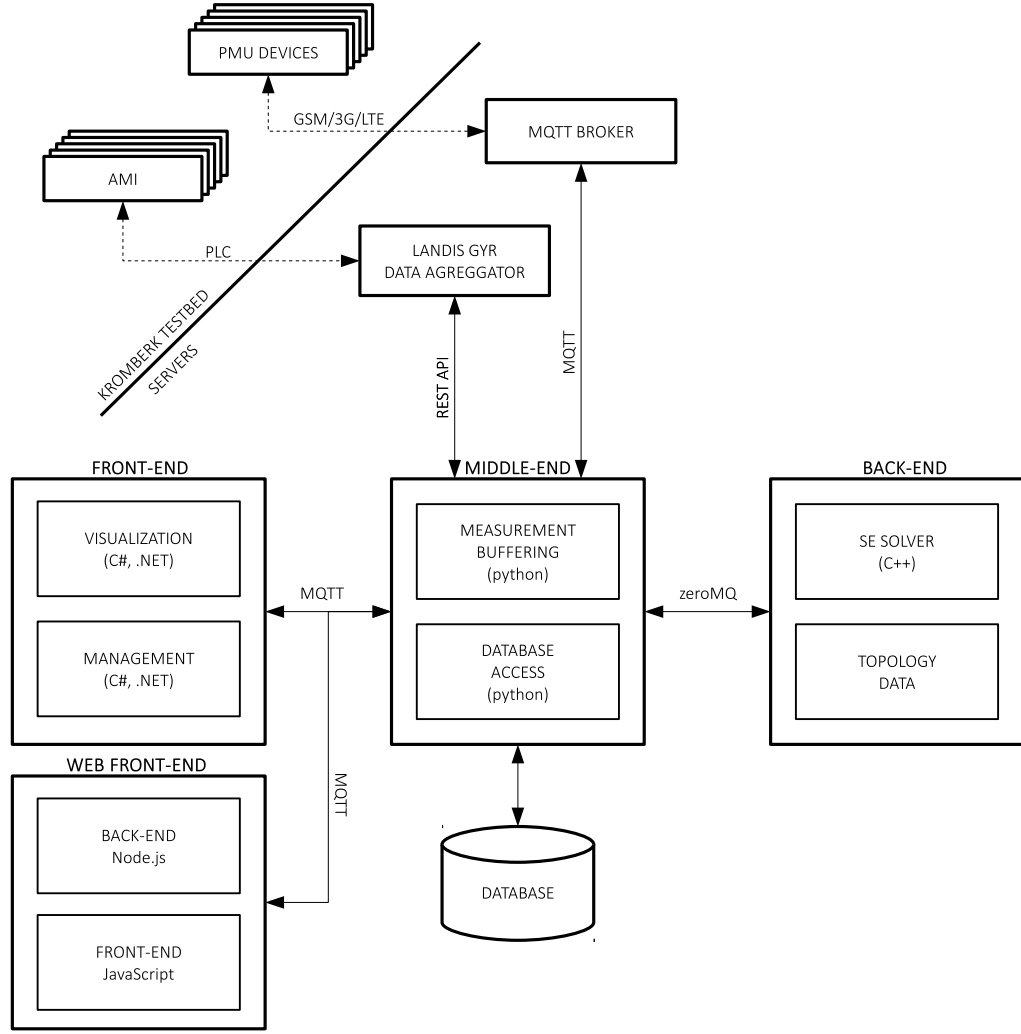


Figure 5.2: Software architecture.

we selected the WLS estimator for state estimation, the bad data detection algorithm was implemented separately in the middle-ware of the solution.

In order to identify the outliers of the measurement cluster in n -dimensions, a measure of distance of each measurement with respect to the bulk of the measurement cluster is needed. One possible measure is provided by the Mahalanobis distance, which makes use of the sample mean and the sample covariance matrix.

The set of m measurements can be defined by the set of n dimensional vectors:

$$\begin{aligned}
 \mathbf{h}_1 &= (h_{11}, h_{12}, \dots, h_{1n}) \\
 \mathbf{h}_2 &= (h_{21}, h_{22}, \dots, h_{2n}) \\
 &\dots \\
 \mathbf{h}_m &= (h_{m1}, h_{m2}, \dots, h_{mn}).
 \end{aligned} \tag{5.1}$$

Each of the m vectors corresponds to the row in the system's Jacobian matrix $\mathbf{H}$. The sample mean of the k -th state variable is then:

$$\bar{\mathbf{h}} = \frac{1}{m} \sum_{i=1}^m \bar{\mathbf{h}}_i. \tag{5.2}$$

The sample covariance matrix is:

$$\mathbf{C} = \frac{1}{m-1} \sum_{i=1}^m (\mathbf{h}_i - \bar{\mathbf{h}})(\mathbf{h}_i - \bar{\mathbf{h}})^T. \quad (5.3)$$

With the established sample mean and sample covariance, the Mahalanobis distance MD_i of a measurement point $\mathbf{h}_i$ is defined as:

$$MD_i = \sqrt{(\mathbf{h}_i - \bar{\mathbf{h}})^T \mathbf{C}^{-1} (\mathbf{h}_i - \bar{\mathbf{h}})}. \quad (5.4)$$

If normal distribution is assumed for measurements, then the MD_i^2 has approximately an χ^2 distribution with n degrees of freedom. Therefore, there is a probability of approximately $1-a$ that a point $\mathbf{h}_i$ will fall inside the tolerance ellipsoid satisfying $MD < \chi^2$. This provides a criterion which tags as outliers all data points whose Mahalanobis distances are larger than a cutoff value, say larger than $\sqrt{\chi_{a,0.975}^2}$. It is necessary to bear in mind that, since the MDi are based on non-robust statistics, they are prone to the masking effect of multiple outliers, especially when the latter appear in clusters [39].

In the next stage of pre-processing phase, time base normalization and time alignment are performed. Normally, measurements are timestamped, but measurement devices can have different reporting rates. Additionally, communication links to measurement devices have different latencies making it necessary to buffer and align them, before passing them to the state estimation engine. For time alignment, we implemented a simple bucket filling scheme depicted in Fig. 5.3.

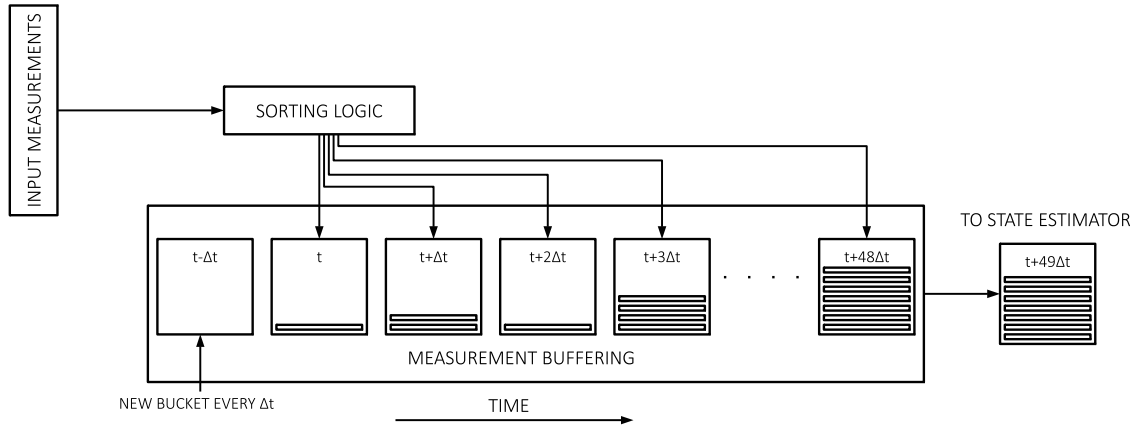


Figure 5.3: Time buffering with bucket filling.

As the measurements come in, they are put into time tagged buckets. The number of buckets must be equal or larger than the greatest expected communication delay. Because the frequency of reports is different for different measurement devices, they must be normalized to the same time base using decimation and interpolation. For example, PMU measurements have high reporting rates (50 measurements per second) comparatively, smart-meters have very low reporting rates with 15 minute reporting interval. Because the state estimation engine is computed every 20 milliseconds, the AMI measurements need to be interpolated to this frequency. Outright interpolation is not possible because the next measurement is not present yet. This is why the next measurement needs to be forecast. The simplest possible forecasting algorithm was selected, which is that we assume no change in values in the next received measurement. The weight of the particular measurement is linearly decreased tenfold to account for loss in precision (Fig. 5.4).

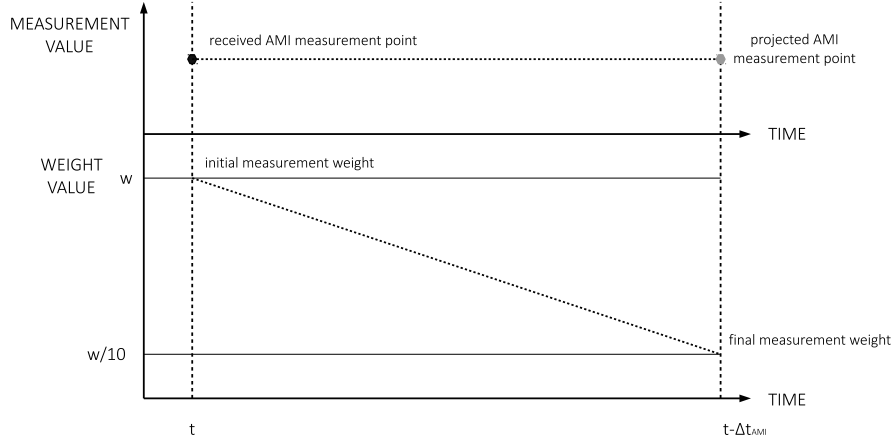


Figure 5.4: Measurement and weight extrapolation.

5.2.2 State estimation solver

The state estimation solver implements a WLS norm and uses Hachtel's method to converge on a solution. The implementation was done in C++, using the Armadillo library for matrix computations [68]. Time aligned measurements are received from the measurement preprocessing module via the zeroMQ publish subscribe protocol. As soon as the measurements are received, the state estimation calculation is triggered. A flat start is used as the initial solution. If the algorithm successfully converges on a solution, it outputs it to the assigned zeroMQ topics. The algorithm needs to finish the computation before the next batch of measurements is received.

5.2.3 Data storage

Raw measurements and state estimation results are stored in the mongoDB database. MongoDB is a noSQL database program that uses the JavaScript Object Notation (JSON) open-standard file format. MongoDB supports field, range query, and regular expression searches [69].

A NoSQL database, or non relational database, provides a mechanism for storage and retrieval of data that is modeled in means other than the tabular relations used in relation databases. NoSQL databases are increasingly used in big data and real-time web applications [70]. The NoSQL database was selected because of simplicity and flexibility of design and simplicity of horizontal scaling i.e. sharding.

A database daemon was written in Python that subscribes to the zeroMQ and MQTT topics and stores the incoming measurements and results.

5.2.4 Visualization

The visualization module was implemented as a web based application in Javascript and nodeJS. The application displays the relevant distribution grid section in the Google Maps API, and can visualize streams of measurement data in pseudo real-time. Fig. 5.5 depicts application block diagram. The web front-end client (user) sends a request for a specific measurement stream. The web back-end responds by subscribing to the requested MQTT topic. The web application back-end parses the received data and sends them to the front-end via web sockets. Web front-end then renders the graph in pseudo real-time [71].

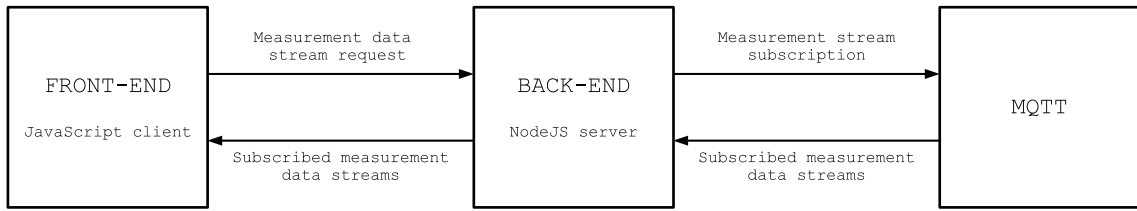


Figure 5.5: Web application block diagram.

Application Graphical User Interface (GUI) is shown in Fig. 5.6. Medium and low voltage networks are depicted in green and blue colors respectively.

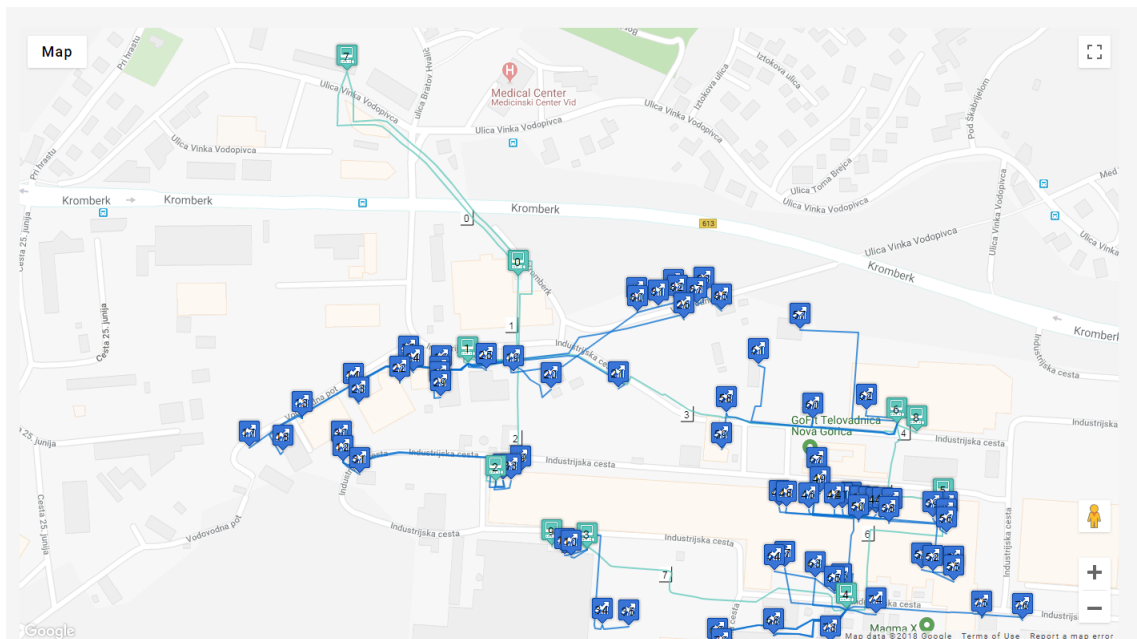


Figure 5.6: Web application graphical user interface.

By selecting a node, the application visualizes the state estimation results or received measurements, if they are available, for the selected bus. In the case of PMU measurements, the displayed quantities are frequency (Fig. 5.9), magnitudes (Fig. 5.7), and phases (Fig. 5.8).

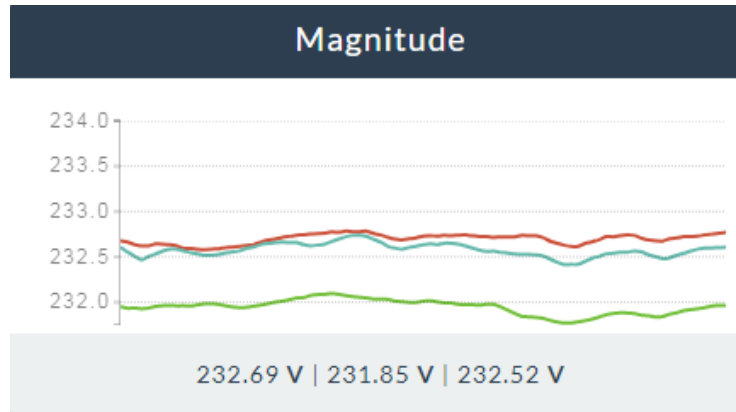


Figure 5.7: Pseudo real-time visualization of three-phase voltage magnitudes.

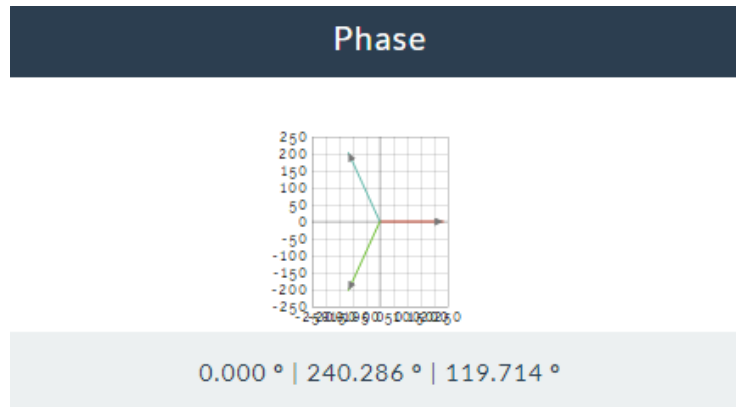


Figure 5.8: Pseudo real-time visualization of three-phase voltage phases.

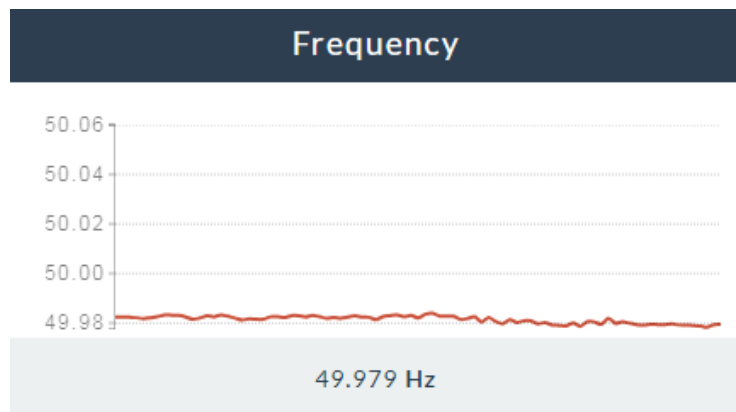


Figure 5.9: Pseudo real-time visualization of frequency.

5.2.5 Communication protocols

For communication between devices and visualization front-end, a Message Queuing Telemetry Transport (MQTT) publish-subscribe based messaging protocol was used. The protocol works on top of the TCP/IP protocol. An MQTT system consists of clients communicating

with a server, called broker. A client can be both a publisher and a subscriber of information. Information flows are organized in a hierarchical topic tree. When a publisher has a new message to distribute, they send it to the broker. The broker then distributes the information to any clients that have previously subscribed to that topic. The publisher does not need any data about the subscribers, and the subscriber, in turn, do not need any data about the publishers. Topic names define the connection [72]. An example of an MQTT connection with connect, publish/subscribe, and disconnect messages is depicted in Fig. 5.10.

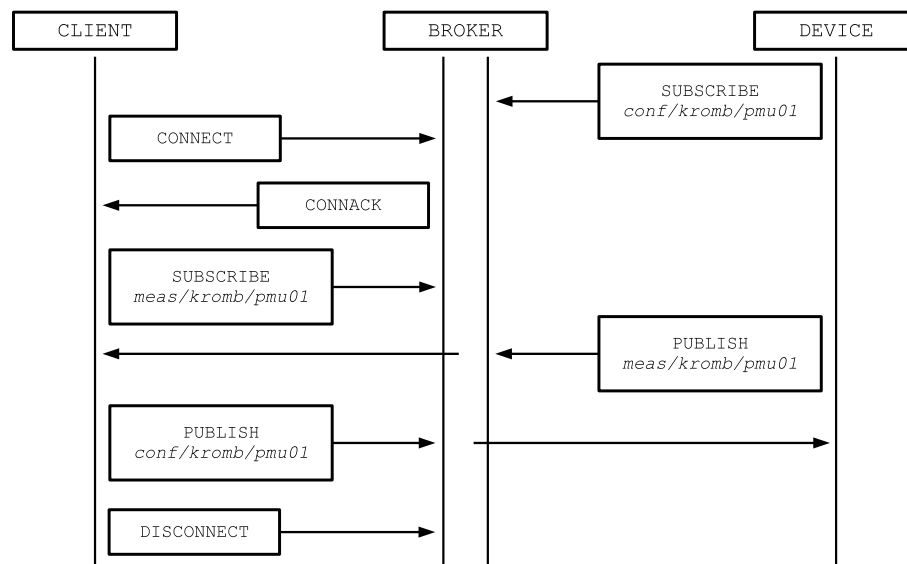


Figure 5.10: Web application block diagram.

Fig. 5.11 depicts a topic tree developed for our SE system. When a client subscribes to a topic, they can subscribe to the exact topic of a published message or they can use wildcards to subscribe to multiple topics simultaneously [73]. This is useful on the middle-end part of the presented SE solution, where the sorting logic needs to capture measurement streams of all deployed devices.

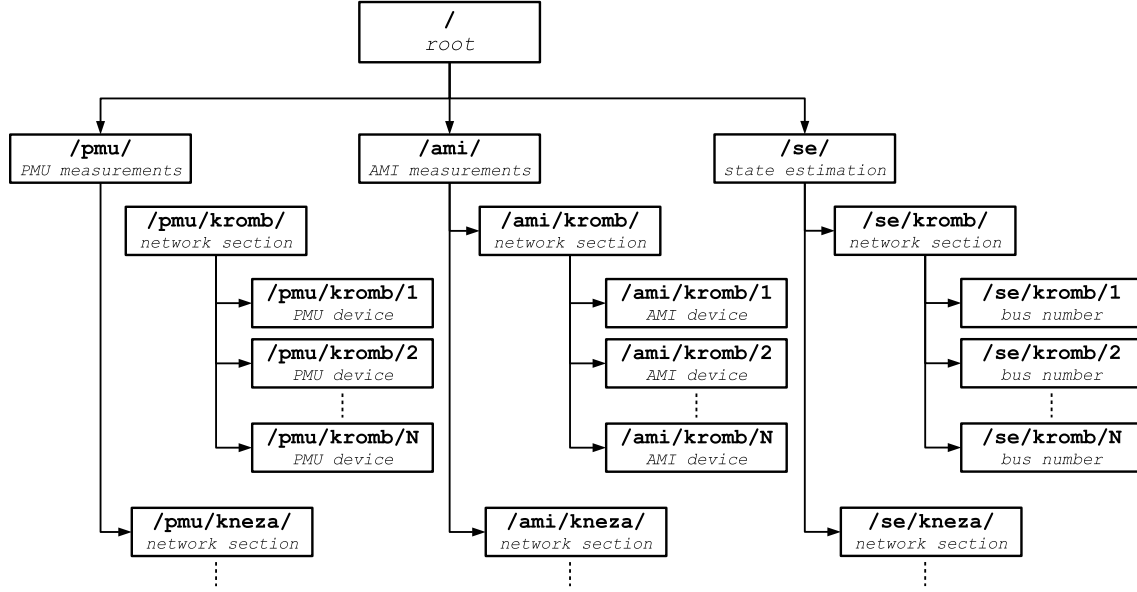


Figure 5.11: MQTT topic tree.

For communication between functional blocks, a ZeroMQ publish-subscribe protocol was used. ZeroMQ is a high-performance asynchronous messaging library, for use in distributed applications. Unlike MQTT, a zeroMQ system can run without a dedicated message broker [74].

5.3 Micro-Synchrophasor Measurement Unit

Synchronized phasor measurements are the measurement technique of choice for transmission power systems. With factors adding to uncertainty and variability in distribution power systems, phasor measurements are also making their way into distribution systems. The key technical difference between distribution and transmission systems affecting phasor measurements are the X/R ratios of lines and cables. The low series inductances in distribution systems cause much smaller phase displacements. This requires a higher resolution of phase angle measurements which, in turn, puts higher demands on synchronization.

In this chapter, a hardware architecture of a small, low-cost μPMU device suitable for distribution networks will be presented. Estimation algorithms for phase, frequency, rate of change of frequency, and quality of phasor estimation will be reviewed. Software architecture for the implementation of the reviewed algorithms in a signal processing device with data acquisition and communication capabilities will be presented.

Some technical details, such as synchronization, error sources, and methods of their compensation, needed for the actual implementation are omitted from the discussion.

5.3.1 Hardware architecture

The μPMU device consists of a digital processing unit, general purpose processor, necessary peripherals, measurement front-end, and clock circuit which supplies a synchronized clock to the sampling subsystem. The digital processing unit is where real-time signal processing is performed (estimation of phasor, frequency, Rate Of Change Of Frequency (ROCOF), and transient monitor) and can be realized with Field Programmable Gate Arrays (FPGA) or Digital Signal Processor (DSP). The general purpose processor provides configuration and communication functionalities.

The crucial part of synchrophasor estimation is availability of the reference time signal. Our measurement accuracy requirement for phase angle measurements was 0.001° (2 sigma). This, in a 50Hz frequency system, determines the total time error budget at 55ns . Factors that draw from the time error budget are clock synchronization and analog measurement front-end. Correct GPS signal manipulation can ensure that time synchronization does not exhaust a significant percentage of the total error budget.

The measurement front-end is another crucial part of the design, since its incorrect design can be a source of phase and magnitude errors. The design includes signal conditioning and an anti-aliasing filter. All of the used components are subjected to tolerances achievable by the production technology process as well as to changing outside factors such as temperature. If, for example, a first order low-pass RC filter is used for anti-aliasing, and the cutoff frequency is set at 2kHz ($R = 1,2\text{k}\Omega$, $C = 68\text{nF}$), and if capacitors are 0.01% unstable in value, this accounts to a $\pm 0.003^\circ$ phase shift at the cutoff frequency. Thus, care must be taken in the selection of components and the filter design phase. Sources of errors must be accounted for and compensated.

Fig. 5.12 depicts the hardware architecture block diagram. Besides the core functions, such as clock circuit, measurement front-end, calibration circuit, and GPS module, the diagram presents all the blocks that are necessary for the operation of such a device. The device is equipped with several communication interfaces: besides the cellular modem, the device features ethernet connectivity and an additional RS232 port that can be used for debugging purposes. The microSD card adds data storage support. Flash memory stores the system's bootloader. The system is powered from the wall adapter that supplies a 9V direct current, but it also has a battery pack that ensures sampling and storage of measurements in the case of power outage.

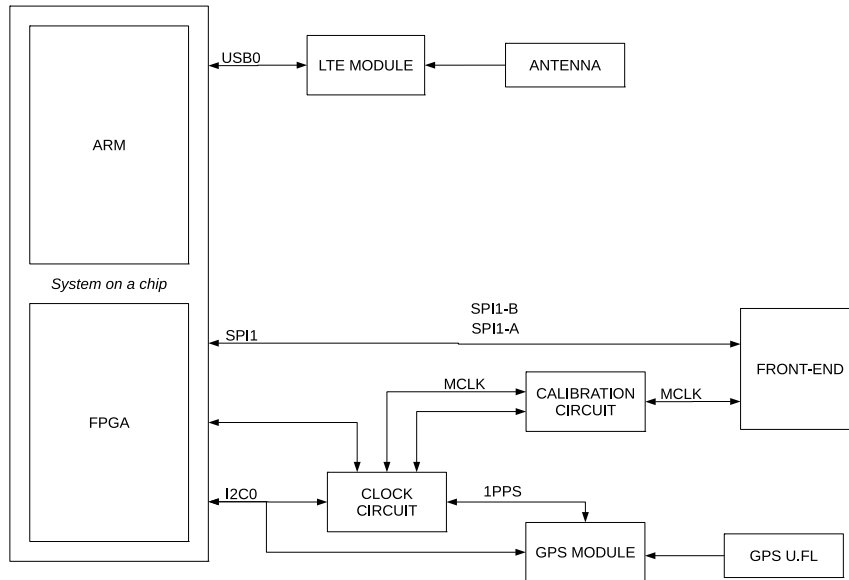


Figure 5.12: PMU Hardware architecture.

5.3.2 Software architecture

The software architecture consists of the functional blocks for filtering, phasor estimation, and frequency estimation.

The most commonly used algorithm for phasor estimation is the single tone Discrete Fourier Transform (DFT) [75]. However, there are several practical considerations that

need to be addressed for a practical implementation to deliver the expected results in a real-world system.

If the harmonic signal is sampled above the Nyquist frequency, its samples can be written as:

$$x_n = X_m \cos\left(n \frac{2\pi}{N} + \Phi\right), \quad (5.5)$$

where N is the number of samples in a single acquired signal period. DFT equations can be written as follows:

$$X_k = \frac{\sqrt{2}}{N} \sum_{n=0}^{N-1} x_n \left(\cos\left(kn \frac{2\pi}{N}\right) - j \sin\left(kn \frac{2\pi}{N}\right) \right) = \frac{\sqrt{2}}{N} \sum_{n=0}^{N-1} x_n e^{j \frac{2\pi kn}{N}}, \quad (5.6)$$

$$X_{kc} = \frac{\sqrt{2}}{N} \sum_{n=0}^{N-1} x_n \cos\left(kn \frac{2\pi}{N}\right), \quad (5.7)$$

$$X_{ks} = \frac{\sqrt{2}}{N} \sum_{n=0}^{N-1} x_n \sin\left(kn \frac{2\pi}{N}\right), \quad (5.8)$$

$$X_k = X_{kc} - jX_{ks}. \quad (5.9)$$

A linear Least Squares (LS) estimator can be constructed for the fundamental phasor component ($k = 1$) using Eqn. 5.6:

$$\mathbf{x} = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ x_{N-1} \end{bmatrix} = \begin{bmatrix} \cos 0 & -\sin 0 \\ \cos \Theta & -\sin \Theta \\ \cos 2\Theta & -\sin 2\Theta \\ \vdots & \vdots \\ \cos(N-1)\Theta & -\sin(N-1)\Theta \end{bmatrix} \cdot \begin{bmatrix} X_r \\ X_{im} \end{bmatrix} + \begin{bmatrix} \epsilon_0 \\ \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_{N-1} \end{bmatrix} = \mathbf{S} \cdot \mathbf{X} + \boldsymbol{\epsilon}. \quad (5.10)$$

The LS solution of the formulated problem is:

$$\hat{\mathbf{X}} = \frac{1}{N} \mathbf{S}^T \mathbf{x}. \quad (5.11)$$

The estimated phasor $\hat{\mathbf{X}}$ can be written as:

$$\hat{X} = \hat{X}_r + j\hat{X}_i. \quad (5.12)$$

Considering that the phasor estimation is a continuous process, it is necessary to perform a calculation every time a new data sample is available. This can be achieved with a sliding window depicted in Fig. 5.13 [75].

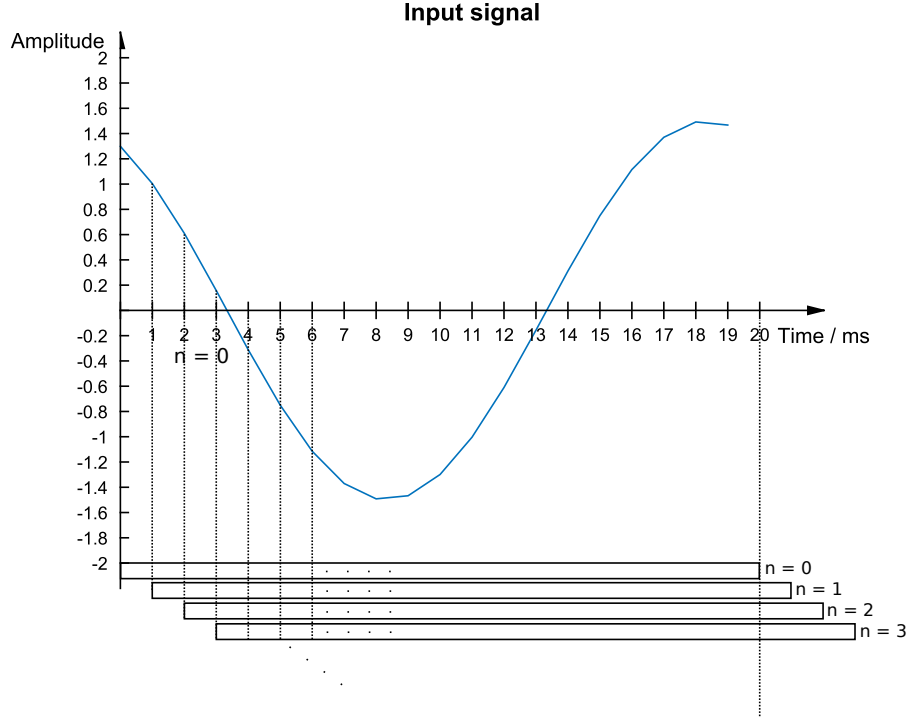


Figure 5.13: Sliding window.

A recursive version of the algorithm which has significantly lower computational requirements can be developed, but it can become unstable in the case of an irregular input signal.

If Gaussian noise contribution with zero mean and σ^2 variance is assumed in input samples, the developed estimator is unbiased, i.e. it yields a true value on average. The covariance matrix of the error in phasor estimate can be written as:

$$E[(\hat{\mathbf{X}} - \mathbf{X})(\hat{\mathbf{X}} - \mathbf{X})^T] = [\mathbf{S}^T \mathbf{S}]^{-1} = \frac{\sigma^2}{N}. \quad (5.13)$$

Thus, a higher number of samples in a calculation window will produce improvements in the estimation variance in inverse proportion of the square root of the number of samples. This can be achieved with increased sample rates of the Analog to Digital Converter (ADC), or by using a longer calculation window. However, longer calculation windows, as a dual property, produce estimates less responsive to the possible dynamic behavior of the input signal.

Since phasors are a steady state concept, the algorithm developed assumes the fundamental frequency at 50 Hz. In reality, a power system is never in a steady state at 50 Hz (Fig. 5.14). A power system normally operates at $\pm 0.5 \text{ Hz}$ from its nominal value.

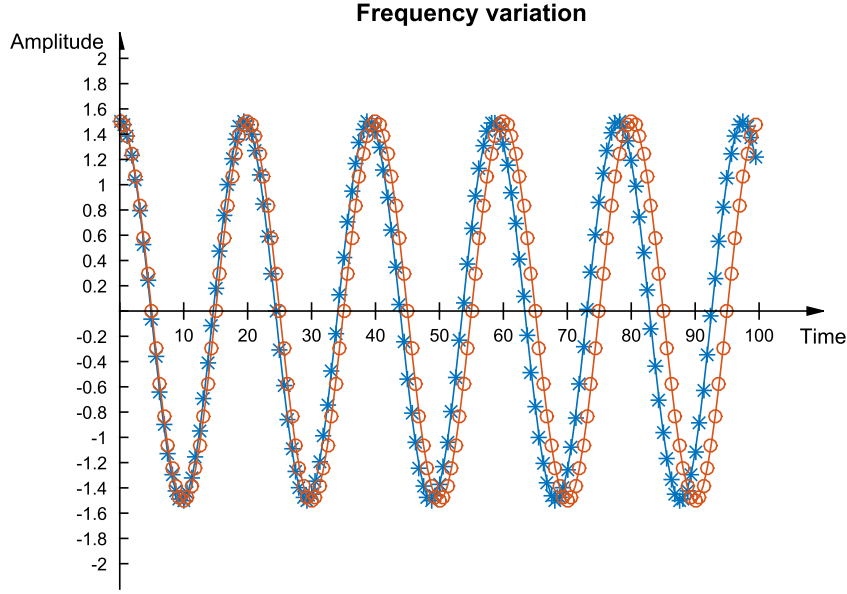


Figure 5.14: Frequency variation.

The variation of frequency causes phasor misestimation. The magnitude of error depends on the difference between nominal and actual frequency. The effect can be quantified and compensated. To first quantify the effect, the input signal is written as [75]:

$$x(t) = X_m \cos(\omega t + \phi) = \sqrt{2} \operatorname{Re} \left[\frac{X_m}{\sqrt{2}} e^{j\phi} e^{j\omega t} \right] = \sqrt{2} \operatorname{Re} [X e^{j\omega t}] \quad (5.14)$$

$$= \frac{\sqrt{2}}{2} (X e^{j\omega t} + X^* e^{-j\omega t}) \quad (5.15)$$

$$x_k = \frac{1}{\sqrt{2}} (X e^{j\omega k \Delta t} + X^* e^{-j\omega k \Delta t}). \quad (5.16)$$

The phasor representation of $x(t)$ is:

$$X'_r = \frac{\sqrt{2}}{N} \sum_{k=r}^{r+N-1} x_k e^{-jk\omega_0 \Delta t} = \frac{1}{N} \sum_{k=r}^{r+N-1} (X e^{jk\omega \Delta t} + X^* e^{-jk\omega \Delta t}) e^{-jk\omega_0 \Delta t}. \quad (5.17)$$

Using identity:

$$e^{jx} - 1 = e^{j\frac{x}{2}} (e^{j\frac{x}{2}} - e^{-j\frac{x}{2}}) = 2j e^{j\frac{x}{2}} \sin \frac{x}{2}, \quad (5.18)$$

and generalized geometric series:

$$\sum_{k=a}^b r^k = \frac{r^a - r^{b+1}}{1 - r}, \quad (5.19)$$

the expression describing phasor estimation at off-nominal frequencies is obtained:

$$X_r = PXe^{jr(\omega-\omega_0)\Delta t} + QX^*e^{-jr(\omega+\omega_0)\Delta t}, \quad (5.20)$$

$$P = \frac{\sin \frac{N(\omega-\omega_0)\Delta t}{2}}{N \sin \frac{(\omega-\omega_0)\Delta t}{2}} e^{j(N-1)\frac{(\omega-\omega_0)}{2}\Delta t}, \quad (5.21)$$

$$Q = \frac{\sin \frac{N(\omega+\omega_0)\Delta t}{2}}{N \sin \frac{(\omega+\omega_0)\Delta t}{2}} e^{j(N-1)\frac{(\omega+\omega_0)}{2}\Delta t}, \quad (5.22)$$

where X_r is the estimated phasor, and P and Q are complex attenuation constants [75]. The result is depicted in Fig. 5.16.

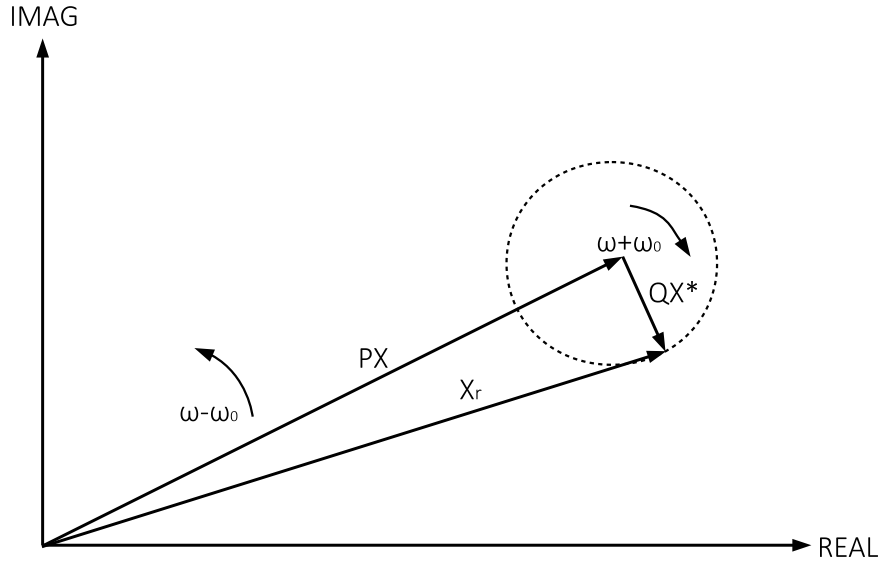


Figure 5.15: Off-nominal frequency phasor estimation.

The QX^* phasor rotates in a negative direction with $\omega + \omega_0 \approx 2\omega_0$. The resultant phasor has a $2\omega_0$ frequency component superimposed to both amplitude and phase as seen in Fig. 5.16.

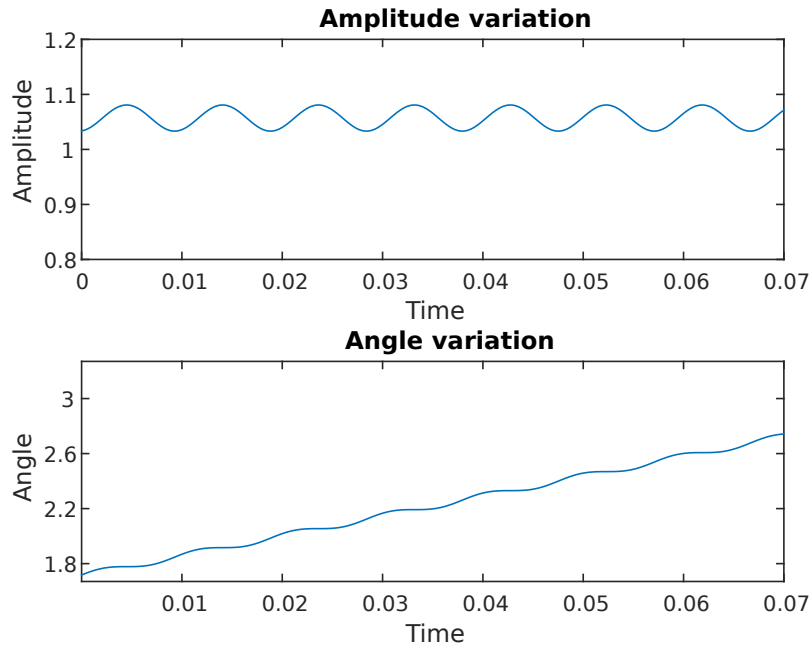


Figure 5.16: Amplitude and phase variation due to off-nominal frequency.

A simple three-point filter that sums up points separated by 120° effectively eliminates the superimposed variation. The result of such filtering is depicted in Fig. 5.17.

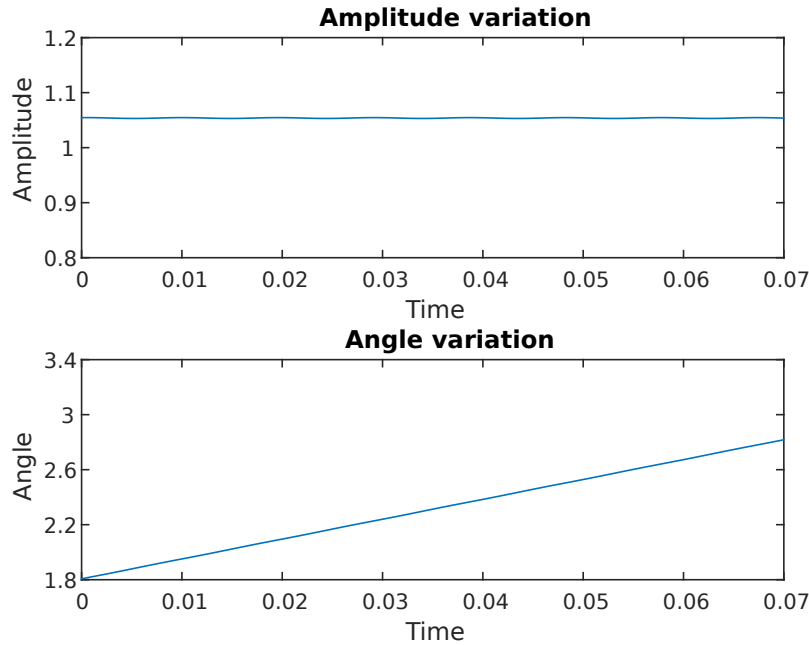


Figure 5.17: Result of three-point filtering.

It can be observed that the second harmonic variation was successfully removed. However, the phasor still rotates in the positive direction with a frequency that is the difference between the actual and nominal frequencies ($\omega - \omega_0$). In terms of Eqn. 5.20, the QX^* term

was removed and the estimated phasor is now:

$$X_r = PXe^{jr(\omega-\omega_0)\Delta t}. \quad (5.23)$$

Besides the information about a signal's phase and magnitude, frequency and ROCOF are parameters of interest. Frequency deviation from the nominal value and the ROCOF for any time can be written as:

$$\omega(t) = (\omega_0 + \Delta\omega + t\omega'). \quad (5.24)$$

A frequency and ROCOF estimator can be constructed using estimated phase, since frequency is the first time derivative of phase [75]. Thus, phase can be written as:

$$\phi(t) = \int \omega dt = \phi_0 + t\omega_0 + t\Delta\omega + \frac{t^2}{2}\omega' = a_0 + a_1t + a_2t^2, \quad (5.25)$$

and the estimator is then constructed as:

$$\phi = \begin{bmatrix} \phi_0 \\ \phi_1 \\ \phi_2 \\ \vdots \\ \phi_{N-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & \Delta t & \Delta t^2 \\ 1 & 2\Delta t & 2^2\Delta t^2 \\ \vdots & \vdots & \vdots \\ 1 & (N-1)\Delta t & (N-1)^2\Delta t^2 \end{bmatrix} \cdot \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \mathbf{B} \cdot \mathbf{A}. \quad (5.26)$$

The solution to the stated linear LS estimation problem is:

$$\mathbf{A} = [\mathbf{B}^T \mathbf{B}]^{-1} \mathbf{B}^T \phi = \mathbf{G} \phi. \quad (5.27)$$

For the estimation of phasor, frequency, and ROCOF matrices, $\mathbf{S}$ and $\mathbf{G}$ can be pre-computed for real-time use.

The reviewed algorithms were architected into a solution depicted in Fig. 5.18.

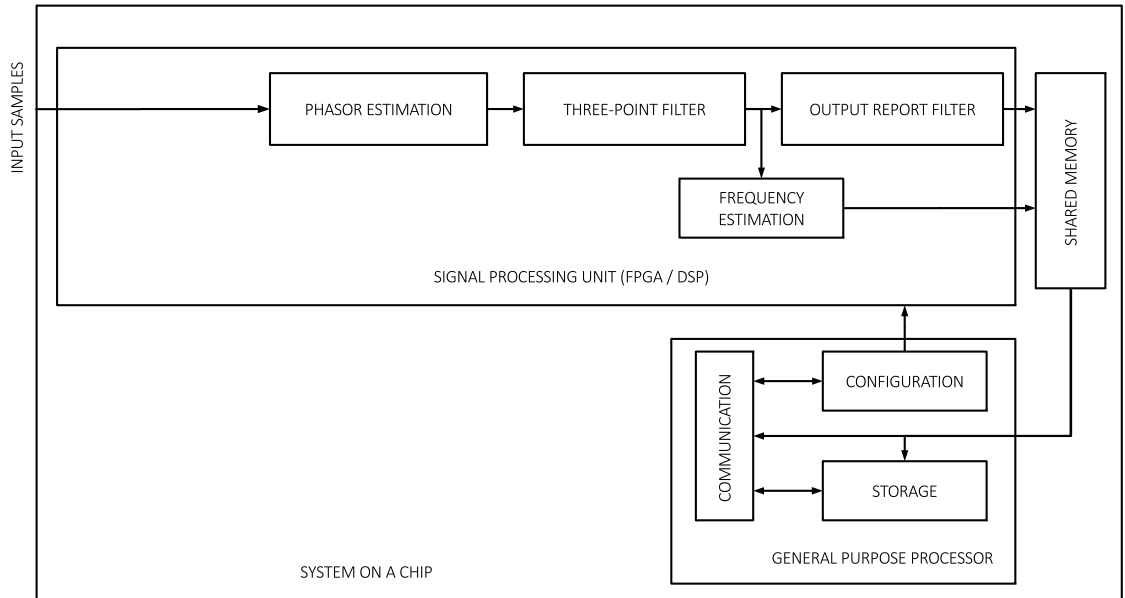


Figure 5.18: Software architecture of the μPMU device.

The General Purpose Processor (GPP) runs a distribution of the Linux operating system that can read the reports from the processing unit and report them via a selected communication protocol. Besides communication capabilities, it also provides configuration and storage capabilities.

5.4 Summary

This chapter investigated the implementation of the SE system to a real-world network. For the modeling phase, a checklist for thorough assesment of the system components was developed. Next, considerations were given with regards to new measurement placement. Next, the developed software architecture was presented. The architecture consists of several loosely coupled functional blocks. Special focus was given to measurement pre-processing since, besides the model, measurements impact the SE results the most.

A concept of a low-cost μPMU device suitable for distribution networks was presented. Specifically, hardware and software architectures were developed. In the hardware architecture system, components were described and major measurement error sources were identified and quantified in terms of total error budget consumption. In the software architecture, the algorithms for the estimation of phasor, frequency, and ROCOF were reviewed. The algorithms implemented as functional blocks were presented in the complete solution.

Chapter 6

Conclusions

Recent changes in power distribution systems will require real-time observation and control of the system to ensure a safe and reliable operation in the future. DSSE presents a crucial part in ensuring observability. It acts like a multi-variate filter of the measurements deployed in the field, capable of outputting the most likely system state. This, in turn, is the information that present and future control functions leverage to ensure a safe and reliable operation of the system.

In this dissertation, the main goal was to develop, implement, and thoroughly evaluate a three-phase DSSE. A unified three-phase branch model that allows a unified modeling of conductors, transformers, tap changers, and voltage regulators was presented. The development of model parameters for all components was reviewed. The problem of statistical state estimation, and how it applies to the estimation of the distribution power system state was also reviewed. A three-phase DSSE that can incorporate heterogeneous measurement types was developed. Several different state estimation algorithms, robust and non-robust, were reviewed and compared based on their statistical efficiency and computational speed. The developed DSSE with all the implemented estimation norms (WLS, SHGM, and LAV) was evaluated for sensitivity of state variables to model and measurement uncertainties. The analysis included different measurement configurations.

Finally, the implemented DSSE was deployed into a real-world network, and the dissertation presented a software architecture with all the necessary support functions. Additionally, a concept of a low-cost μPMU device suitable for distribution power networks was suggested.

6.1 Summary of Contributions

The results presented in this dissertation are expected to contribute to further research and the development of methods, models, and procedures. The developed unified three-phase branch model eases the modeling of the distribution power network. This dissertation includes examples of model applications to conductor sections, transformers, tap changers, and voltage regulators.

The presented sensitivity analysis enables a sensitivity evaluation of the chosen state estimation algorithms and measurement configuration. It was shown that this is highly desirable as it enables an informed choice of the most suitable state estimation algorithm based on the configuration of measurements and the accuracy of the system model. The presented lower and upper interval bounds give clear indication of the performance of the state estimation system that can be expected with the given accuracy of model parameters and measurements.

The introduced concept of a low-cost, high-precision phasor measurement unit includes

hardware and software architectures. As the phase accuracy is very important due to low X/R ratios in distribution systems, the possible error sources are identified and quantified. The device enables many new applications in distribution systems, and is supported in our state estimation data model.

6.2 Future work

The work presented in this dissertation may have several possible research and development directions:

- Large number of buses and consequently measurement devices will create enormous quantities of data that will need to be stored for future manipulation. Technologies that would enable efficient storage and fast recall of measurements and state estimation results will need to be developed. Some research is already underway with the Berkeley Tree Database project [76].
- Further research in current and new applications that leverage state estimation results will be necessary. Applications such as:
 - estimation of impedances and model validation,
 - detection of oscillations,
 - islanding detection and resynchronization,
 - topology detection and real-time estimation of loads,
 - fault detection localization and classification,

and many others, create true customer value in terms of better utilization of available resources, delayed investments into infrastructure, and greater system reliability.

Appendix A

IEEE Distribution Test Feeders

A.1 The 13-bus Feeder

The topology of the IEEE 13-bus feeder is included here. Figure A.1 depicts the one line diagram of the feeder. Labels on branches represent phasing sequences, and labels near spot loads represent load type and its connection (D - delta connection, Y - wye connection, PQ - constant power injection load, Z - constant impedance load, I - constant current load).

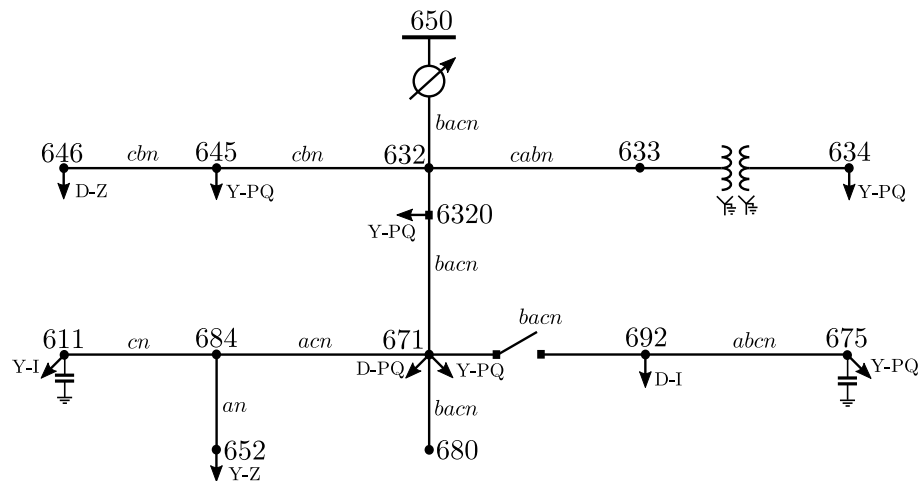


Figure A.1: IEEE 13-bus feeder.

Table A.7 lists the topology data.

Table A.1: IEEE 13 - bus topology data.

Node A	Node B	Length [ft.]	Type	Phasing	Phase No.	Neutral No.	Spacing ID
632	645	500	OHL	CBN	8	8	505
632	633	500	OHL	CABN	6	6	500
633	634	0	Transf.				
645	646	300	OHL	CBN	8	8	505
650	632	2000	OHL	BACN	2	6	500
684	652	800	UGC				
632	671	2000	OHL	BACN	2	6	500
671	684	300	OHL	ACN	8	8	505
671	680	1000	OHL	BACN	2	6	500
671	692	0	Switch				
684	611	300	OHL	CN	8	8	510
692	675	500	UGC				

Table A.2 lists the spot loads on the feeder.

Table A.2: IEEE 13 - spot loads.

Node	Load Type	Ph1 [kW]	Ph1 [kVAr]	Ph2 [kW]	Ph2 [kVAr]	Ph3 [kW]	Ph3 [kVAr]
634	Y-PQ	160	110	120	90	120	90
645	Y-PQ	0	0	170	125	0	0
646	D-Z	0	0	230	132	0	0
652	Y-Z	128	86	0	0	0	0
671	D-PQ	385	220	385	220	385	220
675	Y-PQ	485	190	68	60	290	212
692	D-I	0	0	0	0	170	151
611	Y-I	0	0	0	0	170	80

Table A.3 lists the distributed loads on the feeder.

Table A.3: IEEE 13 - distributed loads.

Node A	Node B	Type	Ph1	Ph1	Ph2	Ph2	Ph3	Ph3
632	671	Y-PQ	17 kW	10 kVAr	66 kW	38 kVAr	117 kW	68 kVAr

Table A.4 lists the shunt capacitors on the feeder.

Table A.4: IEEE 13 - shunt capacitors.

Node	Connection	Ph1 [kVAr]	Ph2 [kVAr]	Ph3 [kVAr]
675	Y	200	200	200
611	Y	0	0	100

Table A.10 lists the transformers on the feeder.

Table A.5: IEEE 13 - transformer.

No.	Node A	Node B	Power	P. vol.	P. conn.	S. vol.	S. conn.	R	X
1	633	634	500 kVA	4.16 kV	Yg	0.48 kV	Yg	1.1 %	2 %

Table A.6 lists the voltage regulators on the feeder.

Table A.6: IEEE 13 - voltage regulators.

Property	Value 1	Value 2	Value 3
Regulator ID	1		
Line segment	650	632	
Location	650		
Phases	A-B-C		
Connection	3-Ph, L-G		
Monitoring phase	A-B-C		
Bandwidth	2	Volts	
Potential transf. ratio	20		
Primary current transf. ratio	700		
Compensator settings	Ph-A	Ph-B	Ph-C
R setting	3	3	9
X setting	9	9	9
Voltage level	122	122	122

A.2 The 123-bus Feeder

The topology of the IEEE 123-bus feeder is included here. Figure A.2 depicts the one line diagram of the feeder.

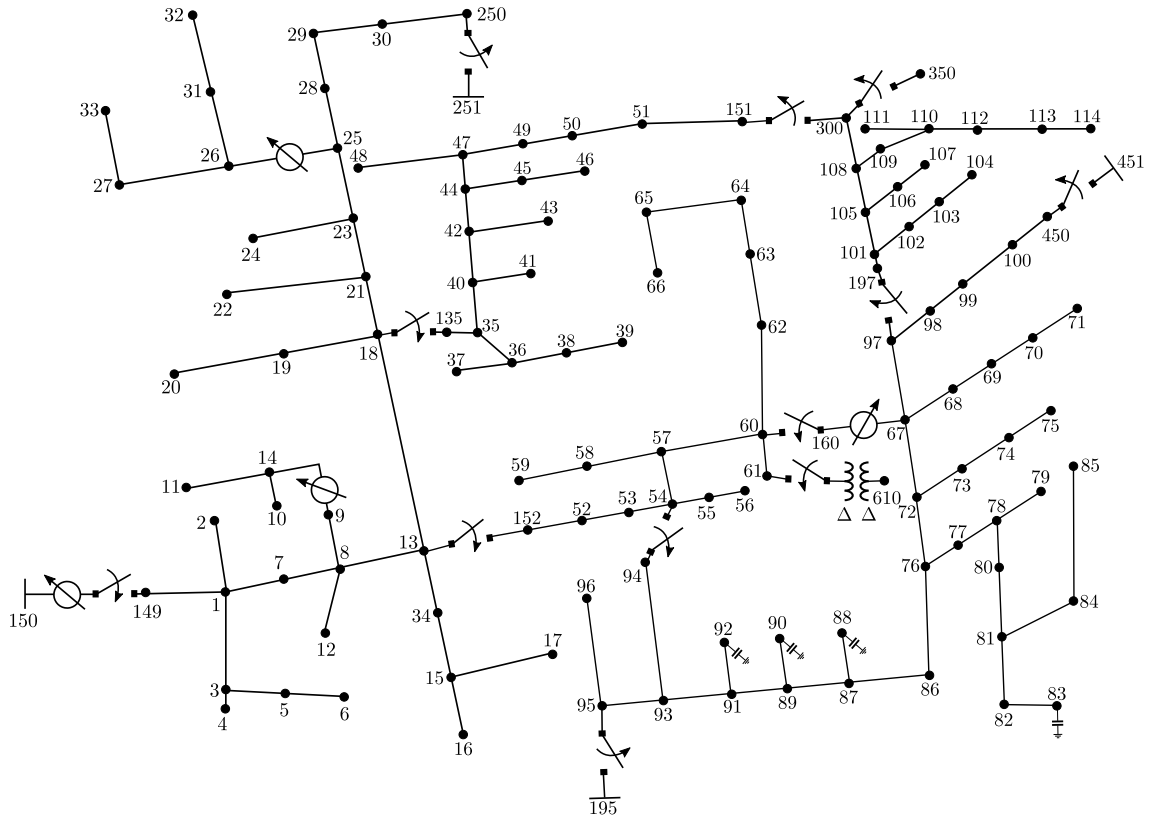


Figure A.2: IEEE 123-bus feeder.

Table A.7: IEEE 123 - bus topology data.

Node A	Node B	Length [ft.]	Type	Phasing	Phase No.	Neutral No.	Spacing ID
1	2	175	OHL	BN	8	8	510
1	3	250	OHL	CN	8	8	510
1	7	300	OHL	ABCN	5	6	500
3	4	200	OHL	CN	8	8	510
3	5	325	OHL	CN	8	8	510
5	6	250	OHL	CN	8	8	510
7	8	200	OHL	ABCN	5	6	500
8	12	225	OHL	BN	8	8	510
8	9	225	OHL	AN	8	8	510
8	13	300	OHL	ABCN	5	6	500
9	14	425	OHL	AN	8	8	510
13	34	150	OHL	CN	8	8	510
13	18	825	OHL	CABN	5	6	500
13	152	0	Switch				
18	135	0	Switch				
14	11	250	OHL	AN	8	8	510
14	10	250	OHL	AN	8	8	510
15	16	375	OHL	CN	8	8	510
15	17	350	OHL	CN	8	8	510
18	21	300	OHL	CABN	5	6	500

Continued on next page

Table A.7 – continued from previous page

Node A	Node B	Length [ft.]	Type	Phasing	Phase No.	Neutral No.	Spacing ID
19	20	325	OHL	AN	8	8	510
21	22	525	OHL	BN	8	8	510
21	23	250	OHL	CABN	5	6	500
23	24	550	OHL	CN	8	8	510
23	25	275	OHL	CABN	5	6	500
25	26	350	OHL	ACN	5	6	505
26	31	225	OHL	CN	8	8	510
27	33	500	OHL	AN	8	8	510
28	29	300	OHL	CABN	5	6	500
29	30	350	OHL	CABN	5	6	500
30	250	200	OHL	CABN	5	6	500
31	32	300	OHL	CN	8	8	510
34	15	100	OHL	CN	8	8	510
35	36	650	OHL	ABN	5	6	505
35	40	250	OHL	ABCN	5	6	500
36	37	300	OHL	AN	8	8	510
36	38	250	OHL	BN	8	8	510
38	39	325	OHL	BN	8	8	510
40	41	325	OHL	CN	8	8	510
40	42	250	OHL	ABCN	5	6	500
42	43	500	OHL	BN	8	8	510
42	44	200	OHL	ABCN	5	6	500
36	37	300	OHL	AN	8	8	510
44	45	300	OHL	AN	8	8	510
44	47	250	OHL	ABCN	5	6	500
45	46	300	OHL	AN	8	8	510
47	48	150	OHL	CBAN	5	6	500
47	49	250	OHL	CBAN	5	6	500
49	50	250	OHL	CBAN	5	6	500
50	51	250	OHL	CBAN	5	6	500
51	151	500	OHL	CBAN	5	6	500
52	53	200	OHL	ABCN	5	6	500
53	54	125	OHL	ABCN	5	6	500
54	55	275	OHL	ABCN	5	6	500
54	57	350	OHL	BCAN	5	6	500
55	56	275	OHL	ABCN	5	6	500
57	58	250	OHL	BN	8	8	510
57	60	750	OHL	BCAN	5	6	500
58	59	250	OHL	BN	8	8	510
60	61	550	OHL	BACN	5	6	500
60	62	250	UGC	ABC	2		515
60	160	0	Switch				
61	610	0	Switch				
61	610	0	XFM-1				
62	63	175	UGC	ABC	2		515
63	64	350	UGC	ABC	2		515
64	65	425	UGC	ABC	2		515

Continued on next page

Table A.7 – continued from previous page

Node A	Node B	Length [ft.]	Type	Phasing	Phase No.	Neutral No.	Spacing ID
65	66	325	UGC	ABC	2		515
67	68	200	OHL	AN	8	8	510
67	72	275	OHL	BCAN	5	6	500
67	97	250	OHL	BCAN	5	6	500
68	69	275	OHL	AN	8	8	510
69	70	325	OHL	AN	8	8	510
70	71	275	OHL	AN	8	8	510
72	73	275	OHL	CN	8	8	510
72	76	200	OHL	BCAN	5	6	500
73	74	350	OHL	AN	8	8	510
74	75	400	OHL	AN	8	8	510
76	77	200	OHL	ACBN	5	6	500
76	86	700	OHL	BCAN	5	6	500
77	78	100	OHL	ACBN	5	6	500
78	79	225	OHL	ACBN	5	6	500
78	80	475	OHL	ACBN	5	6	500
80	81	475	OHL	ACBN	5	6	500
81	82	250	OHL	ACBN	5	6	500
81	84	675	OHL	CN	8	8	510
82	83	250	OHL	ACBN	5	6	500
84	85	475	OHL	CN	8	8	510
86	87	450	OHL	ACBN	5	6	500
87	88	175	OHL	AN	8	8	510
87	89	275	OHL	ACBN	5	6	500
89	90	225	OHL	BN	8	8	510
89	91	225	OHL	ACBN	5	6	500
91	92	300	OHL	CN	8	8	510
91	93	225	OHL	ACBN	5	6	500
93	94	275	OHL	AN	8	8	510
93	95	300	OHL	ACBN	5	6	500
95	96	200	OHL	BN	8	8	510
97	98	275	OHL	BCAN	5	6	500
97	197	0	Switch				
98	99	550	OHL	BCAN	5	6	500
99	100	300	OHL	BCAN	5	6	500
100	450	800	OHL	BCAN	5	6	500
101	102	225	OHL	CN	8	8	510
101	105	275	OHL	BCAN	5	6	500
102	103	325	OHL	CN	8	8	510
103	104	700	OHL	CN	8	8	510
105	106	225	OHL	BN	8	8	510
105	108	325	OHL	BCAN	5	6	500
106	107	575	OHL	BN	8	8	510
108	109	450	OHL	AN	8	8	510
108	300	1000	OHL	BCAN	5	6	500
109	110	300	OHL	AN	8	8	510
110	111	575	OHL	AN	8	8	510

Continued on next page

Table A.7 – continued from previous page

Node A	Node B	Length [ft.]	Type	Phasing	Phase No.	Neutral No.	Spacing ID
110	112	125	OHL	AN	8	8	510
112	113	525	OHL	AN	8	8	510
113	114	325	OHL	AN	8	8	510
135	35	375	OHL	CBAN	5	6	500
149	1	400	OHL	ABCN	5	6	500
150	149	0	Switch				
152	52	400	OHL	ABCN	5	6	500
160	67	350	OHL	ACBN	5	6	500
197	101	250	OHL	BCAN	5	6	500
250	251	0	Switch				
450	451	0	Switch				
54	94	0	Switch				
151	300	0	Switch				
300	350	0	Switch				

Table A.2 lists the spot loads on the feeder.

Table A.8: IEEE 123 - spot loads.

Node	Load Type	Ph1 [kW]	Ph1 [kVAr]	Ph2 [kW]	Ph2 [kVAr]	Ph3 [kW]	Ph3 [kVAr]
1	Y-PQ	40	20	0	0	0	0
2	Y-PQ	0	0	20	10	0	0
4	Y-PQ	0	0	0	0	40	20
5	Y-I	0	0	0	0	20	10
6	Y-Z	0	0	0	0	40	20
7	Y-PQ	20	10	0	0	0	0
9	Y-PQ	40	20	0	0	0	0
10	Y-I	20	10	0	0	0	0
11	Y-Z	40	20	0	0	0	0
12	Y-PQ	0	0	20	10	0	0
16	Y-PQ	0	0	0	0	40	20
17	Y-PQ	0	0	0	0	20	10
19	Y-PQ	40	20	0	0	0	0
20	Y-I	40	20	0	0	0	0
22	Y-Z	0	0	40	20	0	0
24	Y-PQ	0	0	0	0	40	20
28	Y-I	40	20	0	0	0	0
29	Y-Z	40	20	0	0	0	0
30	Y-PQ	0	0	0	0	40	20
31	Y-PQ	0	0	0	0	20	10
32	Y-PQ	0	0	0	0	20	10
33	Y-I	40	20	0	0	0	0
34	Y-Z	0	0	0	0	40	20
35	D-PQ	40	20	0	0	0	0

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Table A.8 – continued from previous page

Node	Load Type	Ph1 [kW]	Ph1 [kVAr]	Ph2 [kW]	Ph2 [kVAr]	Ph3 [kW]	Ph3 [kVAr]
37	Y-Z	40	20	0	0	0	0
38	Y-I	0	0	20	10	0	0
39	Y-PQ	0	0	20	10	0	0
41	Y-PQ	0	0	0	0	20	10
42	Y-PQ	20	10	0	0	0	0
43	Y-Z	0	0	40	20	0	0
45	Y-I	20	10	0	0	0	0
46	Y-PQ	20	10	0	0	0	0
47	Y-I	35	25	35	25	35	25
48	Y-Z	70	50	70	50	70	50
49	Y-PQ	35	25	70	50	35	20
50	Y-PQ	0	0	0	0	40	20
51	Y-PQ	20	10	0	0	0	0
52	Y-PQ	40	20	0	0	0	0
53	Y-PQ	40	20	0	0	0	0
55	Y-Z	20	10	0	0	0	0
56	Y-PQ	0	0	20	10	0	0
58	Y-I	0	0	20	10	0	0
59	Y-PQ	0	0	20	10	0	0
60	Y-PQ	20	10	0	0	0	0
62	Y-Z	0	0	0	0	40	20
63	Y-PQ	40	20	0	0	0	0
64	Y-I	0	0	75	35	0	0
65	D-Z	35	25	35	25	70	50
66	Y-PQ	0	0	0	0	75	35
68	Y-PQ	20	10	0	0	0	0
69	Y-PQ	40	20	0	0	0	0
70	Y-PQ	20	10	0	0	0	0
71	Y-PQ	40	20	0	0	0	0
73	Y-PQ	0	0	0	0	40	20
74	Y-Z	0	0	0	0	40	20
75	Y-PQ	0	0	0	0	40	20
76	D-I	105	80	70	50	70	50
77	Y-PQ	0	0	40	20	0	0
79	Y-Z	40	20	0	0	0	0
80	Y-PQ	0	0	40	20	0	0
82	Y-PQ	40	20	0	0	0	0
83	Y-PQ	0	0	0	0	20	10
84	Y-PQ	0	0	0	0	20	10
85	Y-PQ	0	0	0	0	40	20
86	Y-PQ	0	0	20	10	0	0
87	Y-PQ	0	0	40	20	0	0
88	Y-PQ	40	20	0	0	0	0
90	Y-I	0	0	40	20	0	0
92	Y-PQ	0	0	0	0	40	20
94	Y-PQ	40	20	0	0	0	0
95	Y-PQ	0	0	20	10	0	0

Continued on next page

Table A.8 – continued from previous page

Node	Load Type	Ph1 [kW]	Ph1 [kVAr]	Ph2 [kW]	Ph2 [kVAr]	Ph3 [kW]	Ph3 [kVAr]
96	Y-PQ	0	0	20	10	0	0
98	Y-PQ	40	20	0	0	0	0
99	Y-PQ	0	0	40	20	0	0
100	Y-Z	0	0	0	0	40	20
102	Y-PQ	0	0	0	0	20	10
103	Y-PQ	0	0	0	0	40	20
104	Y-PQ	0	0	0	0	40	20
106	Y-PQ	0	0	40	20	0	0
107	Y-PQ	0	0	40	20	0	0
109	Y-PQ	40	20	0	0	0	0
111	Y-PQ	20	10	0	0	0	0
112	Y-I	20	10	0	0	0	0
113	Y-Z	40	20	0	0	0	0
114	Y-PQ	20	10	0	0	0	0

Table A.9 lists the shunt capacitors on the feeder.

Table A.9: IEEE 123 - shunt capacitors.

Node	Connection	Ph1 [kVAr]	Ph2 [kVAr]	Ph3 [kVAr]
83	Y	200	200	200
88	Y	50	0	0
90	Y	0	50	0
92	Y	0	0	50

Table A.10 lists the transformers on the feeder.

Table A.10: IEEE 13 - transformer.

No.	Node A	Node B	Power	P. vol.	P. conn.	S. vol.	S. conn.	R	X
1	61	150	5000 kVA	115 kV	D	4.16 kV	Yg	1 %	8 %
2		610	150 kVA	4.16 kV	D	0.48 kV	D	1.27 %	2.72 %

Table A.11 lists the voltage regulators on the feeder.

Table A.11: IEEE 123 - voltage regulators.

Property	Value 1	Value 2	Value 3
Property	Value 1	Value 2	Value 3
Regulator ID	1	149	
Line segment	150		
Location	150		
Phases	A-B-C		
Continued on next page			

Table A.11 – continued from previous page

Property	Value 1	Value 2	Value 3
Connection	3-Ph, L-G		
Monitoring phase	A		
Bandwidth	2	Volts	
Potential transf. ratio	20		
Primary current transf. ratio	700		
Compensator settings	Ph-A		
R setting	3		
X setting	7,5		
Voltage level	120		
Regulator ID	2		
Line segment	9	14	
Location	9		
Phases	A		
Connection	1-Ph, L-G		
Monitoring phase	A		
Bandwidth	2	Volts	
Potential transf. ratio	20		
Primary current transf. ratio	50		
Compensator settings	Ph-A		
R setting	0,4		
X setting	0,4		
Voltage level	120		
Regulator ID	3		
Line segment	25	26	
Location	25		
Phases	A-C		
Connection	2-Ph, L-G		
Monitoring phase	A-C		
Bandwidth	1	Volts	
Potential transf. ratio	20		
Primary current transf. ratio	50		
Compensator settings	Ph-A	Ph-C	
R setting	0,4	0,4	
X setting	0,4	0,4	
Voltage level	120	120	
Regulator ID	4		
Line segment	160	67	
Location	160		
Phases	A-B-C		
Connection	3-Ph, L-G		
Monitoring phase	A-B-C		
Bandwidth	2	Volts	
Potential transf. ratio	20		
Primary current transf. ratio	300		
Compensator settings	Ph-A	Ph-B	Ph-C
R setting	0,6	1,4	0,2
X setting	1,3	2,6	1,4
Continued on next page			

Table A.11 – continued from previous page

Property	Value 1	Value 2	Value 3
Voltage level	124	124	124

Table A.12 lists the mixed measurement configuration for validation.

Table A.12: Mixed measurement configuration for validation

Meas. config.	1	2	3	4	5	6	7
Bus / Branch	PMU	PQ	PQ	PQ	PQ	PQ	PQ
1–149	•						•
2–1	•						•
3–1	•						•
4–3	•						•
5–3	•						•
6–5	•						•
7–1	•						•
8–7	•						•
9–8	•						•
10–14	•						•
11–14	•						•
12–8	•					•	
13–8	•					•	
14–9	•					•	
15–34	•					•	
16–15	•					•	
17–15	•					•	
18–13	•					•	
19–18	•					•	
20–19	•					•	
21–18	•					•	
22–21	•					•	
23–21	•				•		
24–23	•				•		
25–23	•				•		
26–25	•				•		
27–26	•				•		
28–25	•				•		
29–28	•				•		
30–29	•				•		
31–26	•				•		
32–31	•				•		
33–27	•				•		
34–13	•				•		
35–135	•				•		

Continued on next page

Meas. config.	1		2	3	4	5	6	7
Bus / Branch	PMU	PQ	PQ	PQ	PQ	PQ	PQ	PQ
36–35	•				•			
37–36	•				•			
38–36	•				•			
39–38	•				•			
40–35	•				•			
41–40	•				•			
42–40	•				•			
43–42	•				•			
44–42	•				•			
45–44	•			•				
46–45	•			•				
47–44	•			•				
48–47	•			•				
49–47	•			•				
50–49	•			•				
51–50	•			•				
52–152	•			•				
53–52	•			•				
54–53	•			•				
55–54	•			•				
56–55	•		•					
57–54	•		•					
58–57	•		•					
59–58	•		•					
60–57	•		•					
61–60	•		•					
62–60	•		•					
63–62	•		•					
64–63	•		•					
65–64	•		•					
66–65	•		•					
67–160	•		•					
68–67	•	•						
69–68	•	•						
70–69	•	•						
71–70	•	•						
72–67	•	•						
73–72	•	•						
74–73	•	•						
75–74	•	•						
76–72	•	•						
77–76	•	•						
78–77	•	•						
79–78	•	•						
80–78	•	•						

Continued on next page

Continued on next page

Table A.12 – continued from previous page

Meas. confg.	1		2	3	4	5	6	7
Bus / Branch	PMU	PQ	PQ	PQ	PQ	PQ	PQ	PQ
81–80	•	•						
82–81	•	•						
83–82	•	•						
84–81	•	•						
85–84	•	•						
86–76	•	•						
87–89	•	•						
88–87	•	•						
89–87	•	•						
90–89	•	•						
91–89	•	•						
92–91	•	•						
93–91	•	•						
94–93	•	•						
95–93	•	•						
96–95	•	•						
97–67	•	•						
98–97	•	•						
99–98	•	•						
100–99	•	•						
101–197	•	•						
102–101	•	•						
103–102	•	•						
104–103	•	•						
105–101	•	•						
106–105	•	•						
107–106	•	•						
108–105	•	•						
109–108	•	•						
110–109	•	•						
111–110	•	•						
112–110	•	•						
113–112	•	•						
114–113	•	•						
135–18	•	•						
149–150	•	•						
151–51	•	•						
152–13	•	•						
160–60	•	•						
197–197	•	•						
250–30	•	•						
300–108	•	•						
450–100	•	•						
610–61	•	•						

Appendix B

Conductor Data

B.1 Conductor Data

Table B.1: Conductor Data.

No.	Conductor size		Stranding	Type	Resistivity	Diameter	GMR	Rating
	Circular miles	AWG						
1	1000000	10.7/0	26/7	AA	0.105	1.15	0.0368	698
2	556500	8.17/0		ACSR	0.186	0.927	0.0311	730
3	500000	7.7/0		AA	0.206	0.813	0.026	483
4	250000	4.7/0		AA	0.41	0.567	0.0171	329
5	336400	6/0	26/7	ACSR	0.306	0.721	0.0244	530
6	211566	4/0	6/1	ACSR	0.592	0.563	0.00814	340
7	133056	2/0		AA	0.769	0.414	0.0125	230
8	105518	1/0		ACSR	1.12	0.398	0.00446	230
9	105518	1/0		AA	0.97	0.368	0.0111	202
10	105518	1/0	6/1	CU	0.607	0.368	0.01113	310
11	66361	2		AA	1.54	0.292	0.00883	156
12	66361	2		ACSR	1.69	0.316	0.00418	180
13	41735	4		ACSR	2.55	0.257	0.00452	140
14	10382	10	6/1	CU	5.903	0.102	0.00331	80
15	6529	12		CU	9.375	0.081	0.00262	75
16	208.080	14		CU	14.872	0.064	0.00208	20

where conductor types are: AA - All Aluminum, ACSR - Aluminum Conductor Steel Reinforced, CU - Copper, and resistance is specified for 60 Hz and 50° Celsius.

B.2 Underground Cables

Table B.2: Underground Concentric Neutral All Aluminum Cable.

Concentric Neutral 15KV All Aluminum Cable - 1/3 Neutral									
No.	Phase conductor size	Number of strands	Diameter over insulation	Diameter over screen	Outside diameter	Neutral conductor size	Number of strands	Neutral material	Ampacity
	Circular miles		Inch	Inch	Inch	Circular miles	AWG		
1	66361	7	0.78	0.85	0.98	208.080	14	CU	135
2	105518	19	0.85	0.93	1.06	208.080	14	CU	175
3	133056	19	0.9	0.97	1.1	208.080	14	CU	200
4	250000	37	1.06	1.16	1.29	208.080	14	CU	260
5	500000	37	1.29	1.39	1.59	6529	12	CU	385
6	1000000	61	1.64	1.77	1.98	10382	10	CU	550

References

- [1] D. Yergin, *The quest: Energy, security, and the remaking of the modern world*. Penguin Publishing Group, 2011, ISBN: 9781101563700.
- [2] M. Pau, “State estimation in electrical distribution systems,” PhD thesis, University of Cagliari, Cagliari, Italy, 2015.
- [3] U. Kuhar, J. Jurše, K. Alič, G. Kandus, and A. Švigelj, “A unified three-phase branch model for a distribution-system state estimation,” in *PES Innovative Smart Grid Technologies Conference Europe (ISGT-Europe), 2016 IEEE*, IEEE, 2016, pp. 1–6.
- [4] U. Kuhar, G. Kosec, and A. Svirgelj, “Measurement noise propagation in distribution-system state estimation,” en, IEEE, May 2017, pp. 1049–1054, ISBN: 978-953-233-090-8. DOI: 10.23919/MIPRO.2017.7973579. [Online]. Available: <http://ieeexplore.ieee.org/document/7973579/> (visited on 07/23/2018).
- [5] U. Kuhar, M. Pantos, G. Kosec, and A. Svirgelj, “The impact of model and measurement uncertainties on a state estimation in three-phase distribution networks,” en, *IEEE Transactions on Smart Grid*, pp. 1–1, 2018, ISSN: 1949-3053, 1949-3061. DOI: 10.1109/TSG.2018.2823398. [Online]. Available: <http://ieeexplore.ieee.org/document/8331939/> (visited on 07/23/2018).
- [6] F. C. Schweppe, “Power system static-state estimation part i: The exact model,” *IEEE Transactions on Power Apparatus and Systems*,
- [7] F. C. Schweppe and B. R. Douglas, “Power system static-state estimation part ii: The approx. model,” *IEEE Transactions on Power Apparatus and Systems*,
- [8] F. C. Schweppe, “Power system static-state estimation, part iii: Implementation,” *IEEE Transactions on Power Apparatus and Systems*, no. 1, pp. 130–135, 1970. (visited on 10/06/2014).
- [9] F. David. (2018). 4 charts that show renewable energy is on the rise in america, [Online]. Available: <https://energy.gov/eere/articles/4-charts-show-renewable-energy-rise-america> (visited on 09/01/2018).
- [10] I. E. A. (IEA). (2018). Global ev outlook, [Online]. Available: https://www.iea.org/publications/freepublications/publication/Global_EV_Outlook_2016.pdf (visited on 09/01/2018).
- [11] G. Haddadian, M. Khodayar, and M. Shahidehpour, “Accelerating the global adoption of electric vehicles: Barriers and drivers,” *The Electricity Journal*, vol. 28, no. 10, pp. 53–68, Dec. 2015, ISSN: 10406190. DOI: 10.1016/j.tej.2015.11.011.
- [12] L. Pieltain Fernandez, T. Gomez San Roman, R. Cossent, C. Mateo Domingo, and P. Frias, “Assessment of the impact of plug-in electric vehicles on distribution networks,” *IEEE Transactions on Power Systems*, vol. 26, no. 1, pp. 206–213, Feb. 2011, ISSN: 0885-8950, 1558-0679. DOI: 10.1109/TPWRS.2010.2049133. (visited on 06/14/2017).

- [13] C. L. Masters, "Voltage rise: The big issue when connecting embedded generation to long 11 kv overhead lines," *Power Engineering Journal*, vol. 16, no. 1, pp. 5–12, 2002. (visited on 05/17/2015).
- [14] M. H. Bollen and N. Etherden, "Overload and overvoltage in low-voltage and medium-voltage networks due to renewable energy - some illustrative case studies," in *2011 2nd IEEE PES International Conference and Exhibition on, Innovative Smart Grid Technologies (ISGT Europe)*, IEEE, 2011, pp. 1–8. (visited on 11/16/2015).
- [15] N. Kreifels, J. N. Mayer, B. Burger, and C. Wittwer, "Analysis of photovoltaics and wind power in future renewable energy scenarios," en, *Energy Technology*, vol. 2, no. 1, pp. 29–33, Jan. 2014, ISSN: 21944288. DOI: 10.1002/ente.201300090. (visited on 06/15/2017).
- [16] A. Oudalov, D. Chartouni, and C. Ohler, "Optimizing a battery energy storage system for primary frequency control," *IEEE Transactions on Power Systems*, vol. 22, no. 3, pp. 1259–1266, Aug. 2007, ISSN: 0885-8950. DOI: 10.1109/TPWRS.2007.901459. (visited on 10/06/2016).
- [17] H.-J. Kunisch, K. G. Kramer, and H. Dominik, "Battery energy storage another option for load-frequency-control and instantaneous reserve," *IEEE Transactions on Energy Conversion*, no. 3, pp. 41–46, 1986. (visited on 10/06/2016).
- [18] T. Borsche, A. Ulbig, M. Koller, and G. Andersson, "Power and energy capacity requirements of storages providing frequency control reserves," in *IEEE PES General Meeting, Vancouver*, Citeseer, 2013. (visited on 10/06/2016).
- [19] Y. Mu, J. Wu, J. Ekanayake, N. Jenkins, and H. Jia, "Primary frequency response from electric vehicles in the great britain power system," *IEEE Transactions on Smart Grid*, vol. 4, no. 2, pp. 1142–1150, Jun. 2013, ISSN: 1949-3053, 1949-3061. DOI: 10.1109/TSG.2012.2220867.
- [20] J. Fan and S. Borlase, "The evolution of distribution," *IEEE Power and Energy Magazine*, vol. 7, no. 2, pp. 63–68, Mar. 2009, ISSN: 1540-7977. DOI: 10.1109/MPE.2008.931392. (visited on 05/20/2015).
- [21] A. Kulmala, A. Mutanen, A. Koto, S. Repo, and P. Jarventausta, "Demonstrating coordinated voltage control in a real distribution network," in *2012 3rd IEEE PES International Conference and Exhibition on, Innovative Smart Grid Technologies (ISGT Europe)*, IEEE, 2012, pp. 1–8. (visited on 11/26/2015).
- [22] R. Caldon, S. Spelta, V. Prandoni, and R. Turri, "Co-ordinated voltage regulation in distribution networks with embedded generation," in *18th International Conference and Exhibition on Electricity Distribution, CIRED 2005.*, IET, 2005, pp. 1–4. (visited on 11/26/2015).
- [23] C. Gao and M. A. Redfern, "A review of voltage control techniques of networks with distributed generations using on-load tap changer transformers," in *2010 45th International Universities Power Engineering Conference (UPEC)*, IEEE, 2010, pp. 1–6. (visited on 11/26/2015).
- [24] M. Meuser, H. Vennegeerts, and P. Schafer, "Impact of voltage control by distributed generation on hosting capacity and reactive power balance in distribution grids," in *Integration of Renewables into the Distribution Grid, CIRED 2012 Workshop*, IET, 2012, pp. 1–5. (visited on 11/16/2015).
- [25] A. Kulmala, S. Repo, and P. Jarventausta, "Active voltage control-from theory to practice," 2012. (visited on 11/16/2015).

- [26] A. Kulmala, K. Maki, S. Repo, and P. Jarventausta, "Active voltage level management of distribution networks with distributed generation using on load tap changing transformers," in *Power Tech, 2007 IEEE Lausanne*, IEEE, 2007, pp. 455–460. (visited on 11/26/2015).
- [27] D. Della Giustina, M. Pau, P. A. Pegoraro, F. Ponci, and S. Sulis, "Electrical distribution system state estimation: Measurement issues and challenges," *IEEE Instrumentation & Measurement Magazine*, vol. 1094, no. 6969/14, 2014.
- [28] C. N. Lu, J. H. Teng, and W.-H. Liu, "Distribution system state estimation," *IEEE Transactions on Power Systems*, vol. 10, no. 1, pp. 229–240, 1995. (visited on 12/07/2014).
- [29] M. E. Baran and A. W. Kelley, "A branch-current-based state estimation method for distribution systems," *IEEE Transactions on Power Systems*, 10th ser., vol. 1995, no. 1, pp. 483–491,
- [30] K. Li, "State estimation for power distribution system and measurement impacts," *IEEE Transactions on Power Systems*, vol. 11, no. 2, pp. 911–916, 1996. (visited on 08/24/2015).
- [31] A. P. Sakis Meliopoulos and F. Zhang, "Multiphase power flow and state estimation for power distribution systems," *IEEE Transactions on Power Systems*, vol. 11, no. 2, pp. 939–946, 1996.
- [32] M. E. Baran and A. W. Kelley, "State estimation for real-time monitoring of distribution systems," *IEEE Transactions on Power Systems*, vol. 9, no. 3, pp. 1601–1609, 1994. (visited on 12/07/2014).
- [33] J. Grainger and W. Stevenson, *Power system analysis*, ser. McGraw-Hill series in electrical and computer engineering: Power and energy. McGraw-Hill, 1994, ISBN: 9780070612938.
- [34] M. A and G. A, "Fast decoupled state estimator,"
- [35] R. Singh, B. Pal, and R. Jabr, "Distribution system state estimation through gaussian mixture model of the load as pseudo-measurement," en, *IET Generation, Transmission & Distribution*, vol. 4, no. 1, p. 50, 2010, ISSN: 17518687. DOI: 10.1049/iet-gtd.2009.0167. (visited on 12/06/2014).
- [36] E. Manitsas, R. Singh, B. C. Pal, and G. Strbac, "Distribution system state estimation using an artificial neural network approach for pseudo measurement modeling," *IEEE Transactions on Power Systems*, vol. 27, no. 4, pp. 1888–1896, Nov. 2012, ISSN: 0885-8950, 1558-0679. DOI: 10.1109/TPWRS.2012.2187804. (visited on 05/20/2015).
- [37] S. M. Kay, *Fundamentals of statistical signal processing: Estimation theory*. Upper Saddle River, NJ, USA: Prentice-Hall, Inc., 1993, ISBN: 0-13-345711-7.
- [38] A. Abur and A. Gomez Exposito, *Power system state estimation: Theory and implementation*. New York: Marcel Dekker, 2004, ISBN: 978-0-8247-5570-6.
- [39] L. Mili, M. G. Cheniae, N. S. Vichare, and P. J. Rousseeuw, "Robust state estimation based on projection statistics [of power systems]," *IEEE Transactions on Power Systems*, vol. 11, no. 2, pp. 1118–1127, 1996.
- [40] W. H. Kersting, *Distribution system modeling and analysis*, ser. The electric power engineering series. Boca Raton: CRC Press, 2002, ISBN: 0-8493-0812-7.
- [41] P. Xiao, D. Yu, and W. Yan, "A unified three-phase transformer model for distribution load flow calculations," en, *IEEE Transactions on Power Systems*, vol. 21, no. 1, pp. 153–159, Feb. 2006, ISSN: 0885-8950. DOI: 10.1109/TPWRS.2005.857847. (visited on 05/25/2015).

- [42] R. Hoffman, "Practical state estimation for electric distribution networks," in *Power Systems Conference and Exposition, 2006. PSCE'06. 2006 IEEE PES*, IEEE, 2006, pp. 510–517.
- [43] P. Janssen, T. Sezi, and J.-C. Maun, "Distribution system state estimation using unsynchronized phasor measurements," in *Innovative Smart Grid Technologies (ISGT Europe), 2012 3rd IEEE PES International Conference and Exhibition on*, IEEE, 2012, pp. 1–6.
- [44] D. A. Haughton and Heydt, "A linear state estimation formulation for smart distribution systems," *IEEE Transactions on Power Systems*, vol. 28, no. 2, pp. 1187–1195, May 2013.
- [45] A. Monticelli, *State estimation in electric power systems: A generalized approach*, ser. The Kluwer international series in engineering and computer science ; Power electronics and power systems SECS 507. Boston: Kluwer Academic Publishers, 1999, ISBN: 978-0-7923-8519-6.
- [46] M. T. Bishop, J. D. Foster, and D. A. Down, "The application of single-phase voltage regulators on three-phase distribution systems," in *Rural Electric Power Conference, 1994. Papers Presented at the 38th Annual Conference*, IEEE, 1994, pp. C2–1.
- [47] A. Gjelsvik, S. Aam, and L. Holten, "Hachtel's augmented matrix method- a rapid method improving numerical stability in power system static state estimation," *IEEE Transactions on Power Apparatus and Systems*, no. 11, pp. 2987–2993, 1985.
- [48] A. Simoes-Costa and V. H. Quintana, "An orthogonal row processing algorithm for power system sequential state estimation," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-100, no. 8, pp. 3791–3800, 1981, ISSN: 0018-9510. DOI: 10.1109/TPAS.1981.317022.
- [49] N. D. Rao and S. C. Tripathy, "Power system static state estimation by the levenberg-marquardt algorithm," *IEEE Transactions on Power Apparatus and Systems*, no. 2, pp. 695–702, 1980. (visited on 08/16/2017).
- [50] I. S. Duff and J. K. Reid, "A comparison of some methods for the solution of sparse overdetermined systems of linear equations," *IMA Journal of Applied Mathematics*, vol. 17, no. 3, pp. 267–280, 1976. (visited on 08/16/2017).
- [51] A. George and M. T. Heath, "Solution of sparse linear least squares problems using givens rotations," *Linear Algebra and its Applications*, vol. 34, pp. 69–83, 1980.
- [52] D. G. Luenberger and Y. Ye, *Linear and nonlinear programming*, 3rd ed, ser. International series in operations research and management science. New York, NY: Springer, 2008, ISBN: 978-0-387-74502-2.
- [53] W. H. Kersting, "Radial distribution test feeders," in *Power Engineering Society Winter Meeting, 2001. IEEE*, vol. 2, IEEE, 2001, pp. 908–912. (visited on 10/16/2015).
- [54] T. A. Stuart and C. J. Herczet, "A sensitivity analysis of weighted least squares state estimation for power systems," *IEEE Transactions on Power Apparatus and Systems*, no. 5, pp. 1696–1701, 1973.
- [55] D. Macii, G. Barchi, and D. Petri, "Uncertainty sensitivity analysis of wls-based grid state estimators," in *IEEE International Workshop on Applied Measurements for Power Systems (AMPS) Proceedings, 2014*, IEEE, 2014, pp. 1–6.
- [56] A. Al-Othman and M. Irving, "Uncertainty modelling in power system state estimation," *IEE Proceedings - Generation, Transmission and Distribution*, vol. 152, no. 2, p. 233, 2005.

- [57] ———, “Analysis of confidence bounds in power system state estimation with uncertainty in both measurements and parameters,” *Electric Power Systems Research*, vol. 76, no. 12, pp. 1011–1018, Aug. 2006.
- [58] C. Rakpenthai, S. Uatrungjit, and S. Premrudeepreechacharn, “State estimation of power system considering network parameter uncertainty based on parametric interval linear systems,” *IEEE Transactions on Power Systems*, vol. 27, no. 1, pp. 305–313, Feb. 2012.
- [59] S. Chakrabarti and E. Kyriakides, “Pmu measurement uncertainty considerations in wls state estimation,” *IEEE Transactions on Power Systems*, vol. 24, no. 2, pp. 1062–1071, May 2009.
- [60] S. Chakrabarti, E. Kyriakides, and M. Albu, “Uncertainty in power system state variables obtained through synchronized measurements,” *IEEE Transactions on Instrumentation and Measurement*, vol. 58, no. 8, pp. 2452–2458, Aug. 2009.
- [61] C. Muscas, S. Sulis, A. Angioni, F. Ponci, and A. Monti, “Impact of different uncertainty sources on a three-phase state estimator for distribution networks,” *IEEE Transactions on Instrumentation and Measurement*, vol. 63, no. 9, pp. 2200–2209, Sep. 2014.
- [62] R. Minguez and A. J. Conejo, “State estimation sensitivity analysis,” *IEEE Transactions on Power Systems*, vol. 22, no. 3, pp. 1080–1091, Aug. 2007.
- [63] E. Castillo, A. J. Conejo, C. Castillo, R. Minguez, and D. Ortigosa, “Perturbation approach to sensitivity analysis in mathematical programming,” *Journal of Optimization Theory and Applications*, vol. 128, no. 1, pp. 49–74, Jan. 2006.
- [64] E. Castillo, *Building and solving mathematical programming models in engineering and science*. Wiley, 2002, ISBN: 978-0-471-15043-5.
- [65] E. Caro, A. Conejo, and R. Minguez, “A sensitivity analysis method to compute the residual covariance matrix,” en, *Electric Power Systems Research*, vol. 81, no. 5, pp. 1071–1078, May 2011, ISSN: 03787796. DOI: 10.1016/j.epsr.2010.12.007.
- [66] R. Singh, B. Pal, and R. Vinter, “Measurement placement in distribution system state estimation,” *IEEE Transactions on Power Systems*, vol. 24, no. 2, pp. 668–675, May 2009, ISSN: 0885-8950, 1558-0679. DOI: 10.1109/TPWRS.2009.2016457.
- [67] R. Singh, B. C. Pal, R. A. Jabr, and R. B. Vinter, “Meter placement for distribution system state estimation: An ordinal optimization approach,” *IEEE Transactions on Power Systems*, vol. 26, no. 4, pp. 2328–2335, Nov. 2011, ISSN: 0885-8950, 1558-0679. DOI: 10.1109/TPWRS.2011.2118771.
- [68] Website. (2018). Armadillo c++ library for linear algebra and scientific computing, [Online]. Available: <http://arma.sourceforge.net/> (visited on 09/01/2018).
- [69] Wikipedia. (2018). MongoDB, [Online]. Available: <https://en.wikipedia.org/wiki/MongoDB> (visited on 09/01/2018).
- [70] ———, (2018). Nosql, [Online]. Available: <https://en.wikipedia.org/wiki/NosQL> (visited on 09/01/2018).
- [71] N. Jokic, “Real-time web-based visualization of distribution electricity grids,” Master’s thesis, Faculty of Computer and Information Science, University of Ljubljana, Ljubljana, Slovenia, 2017.
- [72] Wikipedia. (2018). Mqtt, [Online]. Available: <https://en.wikipedia.org/wiki/MQTT> (visited on 09/01/2018).

- [73] H. .-. website. (2018). Mqtt topics and best practices, [Online]. Available: <https://www.hivemq.com/blog/mqtt-essentials-part-5-mqtt-topics-best-practices> (visited on 09/01/2018).
- [74] Wikipedia. (2018). Zeromq, [Online]. Available: <https://en.wikipedia.org/wiki/zeroMQ> (visited on 09/01/2018).
- [75] A. Phadke and J. Thorp, *Synchronized phasor measurements and their applications*, ser. Power Electronics and Power Systems. Boston, MA: Springer US, 2008, ISBN: 978-0-387-76535-8 978-0-387-76537-2. [Online]. Available: <http://link.springer.com/10.1007/978-0-387-76537-2> (visited on 04/28/2015).
- [76] Website. (2018). Berkeley tree database, [Online]. Available: <http://btrdb.io/> (visited on 09/01/2018).

Bibliography

Publications Related to the Thesis

Journal Articles

- U. Kuhar, M. Pantos, G. Kosec, and A. Svingelj, "The impact of model and measurement uncertainties on a state estimation in three-phase distribution networks," en, *IEEE Transactions on Smart Grid*, pp. 1–1, 2018, ISSN: 1949-3053, 1949-3061. DOI: 10.1109/TSG.2018.2823398. [Online]. Available: <http://ieeexplore.ieee.org/document/8331939/> (visited on 07/23/2018).
- J. J. Nielsen, H. Ganem, L. Jorguseski, K. Alic, M. Smolnikar, Z. Zhu, N. K. Pratas, M. Golinski, H. Zhang, U. Kuhar, Z. Fan, and A. Svingelj, "Secure real-time monitoring and management of smart distribution grid using shared cellular networks," *IEEE Wireless Communications*, vol. 24, no. 2, pp. 10–17, Apr. 2017, ISSN: 1536-1284. DOI: 10.1109/MWC.2017.1600252. [Online]. Available: <http://ieeexplore.ieee.org/document/7909092/> (visited on 07/24/2017).

Conference Papers

- U. Kuhar, J. Jurše, K. Alič, G. Kandus, and A. Švingelj, "A unified three-phase branch model for a distribution-system state estimation," in *PES Innovative Smart Grid Technologies Conference Europe (ISGT-Europe), 2016 IEEE*, IEEE, 2016, pp. 1–6.
- U. Kuhar, G. Kosec, and A. Svingelj, "Measurement noise propagation in distribution-system state estimation," en, IEEE, May 2017, pp. 1049–1054, ISBN: 978-953-233-090-8. DOI: 10.23919/MIPRO.2017.7973579. [Online]. Available: <http://ieeexplore.ieee.org/document/7973579/> (visited on 07/23/2018).
- B. Kazic, S. M., P. Skraba, D. Mladenic, K. Alic, N. R., and A. Svingelj, "Pseudo-measurements based on the forecasted smart meter consumption for state-estimation," in *EMENDER 2015 - energy managemENT Data ElaboRation. Ljubljana: Jozef Stefan Institute*, 2015.
- U. Kuhar, K. Alic, M. A., R. Novak, and A. Svingelj, "Low-voltage-grid state estimation testbed," in *Zbornik stiriindvajsete mednarodne Elektrotehnske in racunalniske konference ERK 2015, 21. - 23. september 2015, Portoroz, Slovenija*, 2015.
- B. Kazic, M. Skrjanc, S. P., K. Kenda, D. Mladenic, U. Kuhar, K. Alic, R. Novak, A. Svingelj, and J. Jurse, "Smart meter consumption prediction for the distributin grid observability analysis," in *Workshop on Smart grid Observability and Control, 8 March 2016, The Hague, The Netherlands*, 2016.
- A. Svingelj, B. Kazic, K. Alic, M. Skrjanc, and U. Kuhar, "State estimation : The key to distribution anagement system," in *Workshop on Advanced Control, Communications and Algorithms for the Smart Grid, 25 March 2015, Aalborg, Denmark*, 2015.

Other Publications

Journal Articles

- A. Hrovat, G. Kandus, U. Kuhar, A. Kelmendi, and A. Vilhar, "A ka-band satellite beacon receiver for propagation experiment," *Informacije MIDE*M, vol. 46, no. 1, pp. 13–23, 2016.

Conference Papers

- S. Mrak, U. Kuhar, and A. Vilhar, "Low-cost system design for tracking satellites in geosynchronous orbit," in *2015 9th European Conference on Antennas and Propagation, EuCAP 2015*, 2015.
- A. Vilhar, G. Kandus, A. Kelmendi, U. Kuhar, A. Hrovat, and M. Schoenhuber, "Three-site ka-band diversity experiment performed in slovenia and austria," in *2015 9th European Conference on Antennas and Propagation, EuCAP 2015*, 2015.
- U. Kuhar, G. Kandus, and A. Vilhar, "Low-cost frequency-stable beacon receiver based on software defined radio," in *2012 20th International Conference on Software, Telecommunications and Computer Networks, SoftCOM 2012*, 2012.
- A. Vilhar, A. Hrovat, S. Mrak, U. Kuhar, and G. Kandus, "Four channel alphasat ka- and q- band beacon receiver station," in *22nd Seminar on Radio Communications, Ljubljana, 3-5 February 2016*, 2016.
- A. Vilhar, A. Hrovat, U. Kuhar, and G. Kandus, "Four channel alphasat ka- and q- band receiver station in ljubljana," in *Satellites and the internet: 2014 conference*, 2014.
- A. Vilhar and U. Kuhar, "The design of a low-cost beacon receiver based on software defined radio," in *Zbornik enaindvajsete mednarodne Elektrotehniške in računalniške konference ERK 2012, 17.-19. september 2012, Portoroz, Slovenija*, 2012.

Biography

Urban Kuhar is a researcher at the Department of Communication Systems, Jožef Stefan Institute, Ljubljana, Slovenia, and a PhD student at the Jožef Stefan International Postgraduate School. He completed his diploma thesis in electrical engineering in 2012 at the Faculty of Electrical Engineering, University of Ljubljana, Slovenia. In 2013, he joined the Department of Communication Systems at the Jožef Stefan Institute. First, he worked in the field of high-frequency satellite communications, where he designed a Ka-band satellite beacon receiver within the ESA SatProSi project. In 2014, he started working on the FP7 SUNSEED project and enrolled in the PhD program. His research interests are related to digital signal processing, statistical signal processing techniques, and power distribution systems. For the past couple of years, he has been active in the field of state estimation algorithms for power distribution systems.